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ANALYSIS AND DESIGN

ROTARY-TYPE SETBACK LEAF S&A MECHANISMS

William E. Ryan

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H•D•L

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a parameter of acceleration function

$A(t)$ time-varying acceleration

A constant acceleration

$\bar{A}$ average acceleration

A_o coefficient of acceleration function

b parameter of acceleration function

c parameter of acceleration function

$$C_1 = 1 - \mu \left[\frac{r}{y} + \frac{m_{i+1}}{m_i} \left(1 + \frac{r}{R}\right) \cos \alpha\right]$$

$$C_2 = 1 - \mu\left(1 + \frac{r}{R}\right)$$

d diameter of spring wire

D mean diameter of coil

E Young's Modulus

f function $f(t)$

F Laplace transform $F(s)$; force

g acceleration due to gravity

h step function

H height of coil

i running index

I moment of inertia

j running index

k radius of gyration of leaf

L Laplace Transform

m mass

M torque

n number of spring coils

N "equivalent" spring torque acceleration

p ratio of practical to absolute velocity change, $\frac{V_p}{V_\delta}$

List of Symbols, cont'd

P	coefficient $\frac{yg\ A_o}{k^2}$
q	coefficient
Q	coefficient $\frac{M_o}{mk^2}$
r	radius of leaf bearing shaft
R	radius of leaf
s	transform variable
S	spring stress
t	time
t_o	initial leaf release time
t_r	time at which leaf is armed
T	arming time period $(t_r - t_o)$
V	velocity
W	weight of leaf
y	distance from axis of rotation to center of mass of leaf
z	relative change in torque
Z	drop-safety index
α	initial angle of leaf position
β	natural frequency of oscillation
δ	impulse, or delta, function
η	ratio $\frac{b}{a}$
θ	leaf rotation angle
λ	spring constant
μ	average coefficient of friction
ρ	density
τ	analog computer machine time

ABSTRACT

A procedure is described for designing S & A (safety-and-arming) setback mechanisms of the rotary-leaf and spring type to obtain optimum safety and reliability. The design procedure, developed from a mathematical analysis, is described in detail. This procedure is formulated so that little mathematical background is required to understand or use it.

An analytical study is made of the effect of varying the mass of the leaves upon the accidental drop safety of the mechanism. The drop safety is defined as the absolute minimum velocity change that can possibly arm the device. This figure of merit is derived and evaluated for each set of leaves.

To determine whether the mechanism arms when fired, graphical curves are developed, showing the arming time of each leaf of different weight as a function of the time that elapses from the moment of firing to their release. By adding these arming times together, the combinations of weights that will arm are readily obtained. This study was made for both fast- and slow-rising acceleration curves, and for restraining springs with both flat and steep spring rates. A simple exponential function, with one varying parameter, was developed to represent different gun acceleration curves.

The results of the analysis in each case indicate that leaves of equal weight are about as safe and reliable as combinations of leaves of varying weights. It was also discovered that the mechanism performed much as though a constant acceleration had been applied to it. The main assumption of the analysis was that friction has an average effective value.

1. INTRODUCTION

The primary S&A (safety-and-arming) mechanism of high-velocity non-rotating ordnance projectiles (artillery or mortar) that are launched with high accelerations is often a multiple-leaf setback device. The pendulum-like leaves of this device, impelled by the firing acceleration, are designed to operate sequentially, one after the other. Each leaf must rotate through a certain angle before the next one is released. The last leaf arms the projectile by releasing a rotor, closing a switch, starting a clock, or some such mechanism. This arrangement insures that the device will arm only if the acceleration lasts a certain finite time and provides safety against accidental handling accelerations of short duration.

Although the leaves are not limited to rotary motion, these are the only type that will be considered in this report, since they are the most common. Also, it will be assumed that the device has three leaves, as in most models, and that they are of flat pendulum configuration mounted on shafts that rotate on bearings. The movement of each leaf under acceleration is opposed by a stiff helical spring mounted under tension on the shaft with one end anchored to the frame and the other to the leaf. A post, which is part of the frame, prevents each leaf from rotating in a negative direction. It is assumed that the leaves are positioned so that they are most sensitive to accelerations in the direction of firing. This three-leaf setback device is a simple inexpensive mechanism, but a design providing the maximum safety and reliability required of ordnance items is rather complicated. The design of most models has relied on experimental testing programs and past performance.

Theoretical and experimental studies conducted by Hausner have added much to the understanding of this device (ref 1,2,3,4). This theoretical study is based on his earlier work. Among the results of Hausner's theoretical analysis that pertain particularly to this study are his derivations of the equations of motion of the leaves, expressions for the principal friction torques on the leaves, and derivations of the absolute minimum velocity change (time integral of acceleration) that the setback mechanism must receive in order to arm. His mathematical model for this last derivation assumes that the spring torque opposing rotation is constant (rather than linear), that there is no friction, and that shock and vibration effects are negligible. A further study of this device was desired to obtain a better understanding of the factors determining the safest and most reliable operation, and to derive a simple procedure for designing these devices--a handbook technique if possible.

The results of this analysis include the recommendation of a simple function to represent various gun accelerations curves. Its shape is changed by varying just a single parameter. A definition

of drop safety is borrowed from Hausner, and a drop-safety index (or figure of merit) is derived for a leaf, which is a function of the spring stiffness and the mass of the leaf. This drop-safety index is proportional to the absolute minimum velocity change that will arm a leaf. Instead of the constant spring torque used by Hausner, a linear spring torque is used in all derivations in this report, so that the effect of the mass-spring system's natural frequency of oscillation is included in the solutions.

Another result of this study is the derivation of an analog computer method by which curves can be obtained, showing the variation in arming time of a leaf as a function of the time that elapses from the application of the gun acceleration until the leaf is released. An analysis of these curves obtained for different spring stiffnesses, leaf masses, and gun acceleration curves shows that there is little increase in drop safety to be gained by using leaves of unequal mass. Therefore, it is recommended that, where possible, leaves be of the same mass. (If the friction load of the latch or rotor on the last leaf is very large, this may not be desirable.) A simple procedure requiring little mathematical background is described in section 3 for designing a leaf-spring setback mechanism that will have the highest drop-safety index and still arm the mechanism.

The technique employed in the analysis and the assumptions made are described in section 2. In this study, any effects due to vibration and shock, such as deformation of parts of the mechanism, are excluded. Also ignored are the weights of the springs and shafts.

2. METHOD OF ANALYSIS

2.1 Derivation of Equations of Motion

A setback leaf is essentially a rotational mass-spring system, which is energized for a very short period of time by a large acceleration during the firing of an artillery device. The setback leaves are designed to "recognize" a particular acceleration function. Each leaf of the mass-spring system absorbs enough energy from the applied acceleration during its existence to rotate to a position that will release the next leaf. These movements of the leaves occur sequentially, and the arming of each leaf must occur in a time short enough to allow the succeeding leaves to accomplish their function also. Since the last leaves are delayed for a period of time before being released, they must be designed for the acceleration that is still available during their travel. In other words, the leaf design depends on the magnitude and shape of the acceleration-time function.

However, a satisfactory design must not only arm for a specified acceleration but must also provide the maximum safety against accidental arming resulting from acceleration-time functions of shapes different

from the operating acceleration. The relative safety of leaves is usually expressed in terms of the velocity change resulting from accidental drops required to arm the leaves. The problem of determining the safety of a leaf and its definition has been discussed by Hausner (ref 3). The measure of the safety of a leaf is chosen to be the absolute minimum velocity change required to arm the leaf. This minimum velocity change will be that resulting from what, in mathematical terminology, is known as an impulse function, or a delta function--a very sharp pulse of infinite acceleration lasting for an infinitely short period of time. Such a pulse is, of course, physically impossible; but it is the limiting case of a high amplitude pulse of short duration, whose velocity change or time integral of acceleration, remains constant.

The equation of motion of a pendulum-type setback leaf rotating about a fixed axis is

$$I \frac{d^2\theta}{dt^2} = \sum_j M_j$$

when M_j are the external torques about the fixed axis of rotation. If the leaf is energized or shocked by an acceleration g A(t), the equation of motion is (ref 2)

$$I \frac{d^2\theta}{dt^2} + M_s + M_f = mg\ A(t)\ y \cos(\theta - \alpha) \qquad (1)$$

where M_s and M_f are the torque of the restraining spring, and the assorted friction torques, respectively.

The particular setback leaf system analyzed is shown in figure 1. It is assumed in the analysis that the acceleration imparted to the system is in the vertical direction. The symbols m, y, and g are the mass of the leaf, the torque arm (distance from the center of mass to the center of the shaft about which the leaf rotates), and the acceleration due to gravity, respectively. Calculations for the center-of-mass and moment of inertia of leaves of this geometry are given in appendix A. The θ is the angle of rotation of the leaf measured as the angle through which the center of mass rotates, assuming that its initial value is zero and that the angle to which it must rotate to arm (or permit the next leaf to rotate) is $\theta - \theta_r$. The axis from which θ is measured is offset by an angle α from the direction perpendicular to the applied acceleration. This angle α is often taken to be $\theta_r/2$. Then as the leaf rotates, $\theta - \alpha$ varies from $-\frac{\theta_r}{2}$ through zero to $+\frac{\theta_r}{2}$. In this analysis

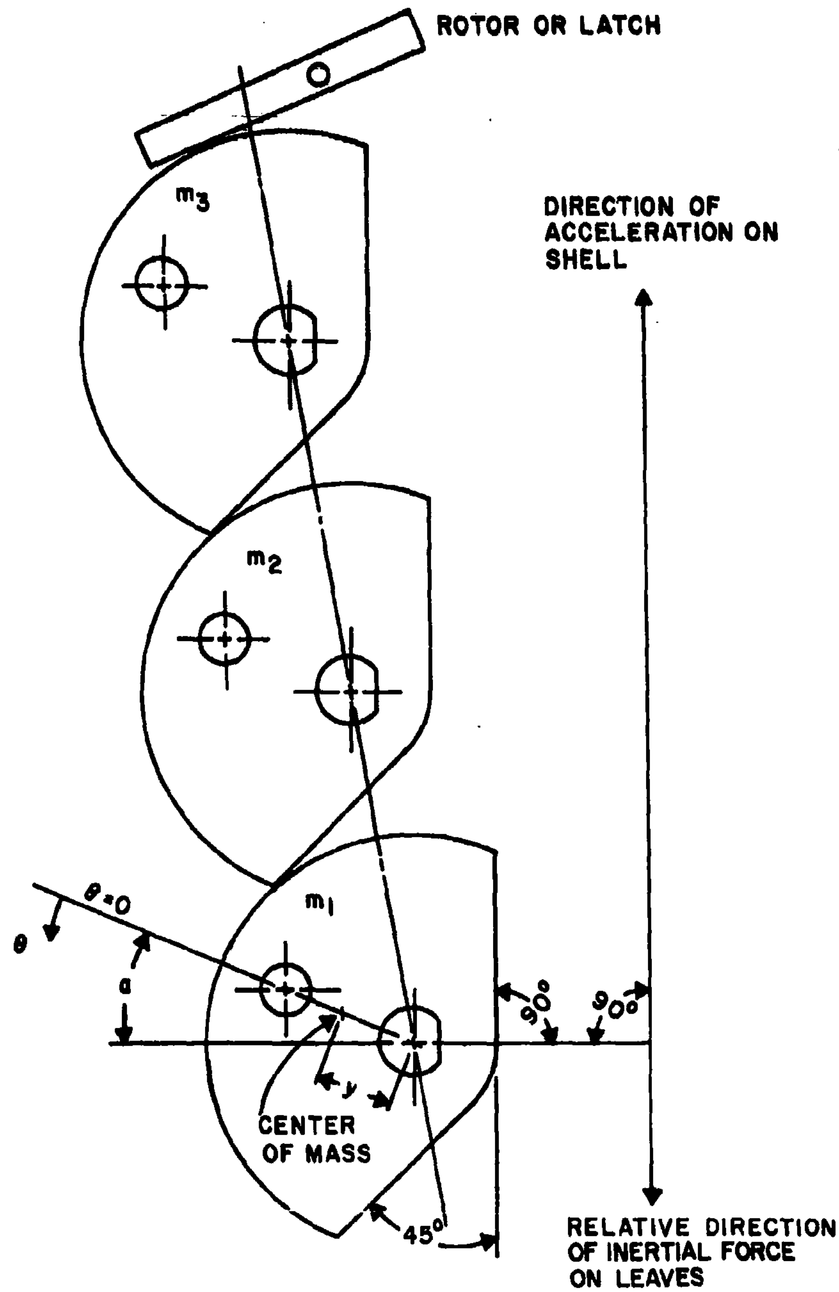

Figure 1. Leaf arrangement in setback mechanism.

Θ_r is 45 deg, so that -22.5 deg $\leq \theta - \alpha \leq$ +22.5 deg. The cos($\theta - \alpha$) factor then varies from 0.934 to unity and back to 0.934. Therefore, there is an error of only a few percent in setting cos($\theta - \alpha$) equal to unity, which reduces a nonlinear equation to a linear approximation. The cos ($\theta - \alpha$) may be considered as a factor modifying the applied acceleration A(t) slightly at the beginning and end of the rotation of each leaf (fig. 2). Since θ is close to zero most of the time as the leaf gathers energy to rotate, there would be actually less reduction in A(t) if α were smaller than $\Theta_r/2$, so that $\theta - \alpha$ would be nearly zero during most of the time that energy is being absorbed by the leaf. However, for this analysis α will be half Θ_r. (If available space is so limited that the leaves cannot be oriented in the same vertical direction as in figure 1, angle α may be different for each leaf.) A(t) is the acceleration in "g", or units of gravity--a dimensionless function of time. The torque of the helical spring opposing the rotation of the leaf is assumed to be a linear function of the angle of rotation θ.

$$M_s = M_o + \lambda\theta = \lambda(\theta_o + \theta) \qquad (2)$$

where M_o is the initial torque on the spring restraining the leaf until the external acceleration is applied; and λ is the spring constant, the rate of increase in torque with the increase in θ. The initial torque M_o is obtained by winding the spring through an initial angle θ_o before assembly. Then as the leaf rotates, the spring is wound through a further angle θ.

The friction torques are caused by pressure on a leaf resisting its rotation. The main friction torques result from pressure on each leaf by the following leaf in the sequence, and from each leaf pressing on its own bearing. The force of any leaf on its predecessor depends on its own torque. This force, to a first approximation (neglecting its own friction torques), is equal to [see fig 3].

$$F_{i+1} = \frac{1}{R_{i+1}} \left[m_{i+1}\, y_{i+1}\, g\, A(t) \cos \alpha_{i+1} - M_{o,\, i+1} \right] \qquad (3)$$

where the subscripts i+1 refer to the (i+1)leaf effect on the (i)th leaf.

The consequent friction torque is

$$M_{Ri} = \mu_R \frac{R_i}{R_{i+1}} \left[m_{i+1}\, y_{i+1}\, g\, A(t) \cos \alpha_{i+1} - M_{o,\, i+1} \right] \qquad (4)$$

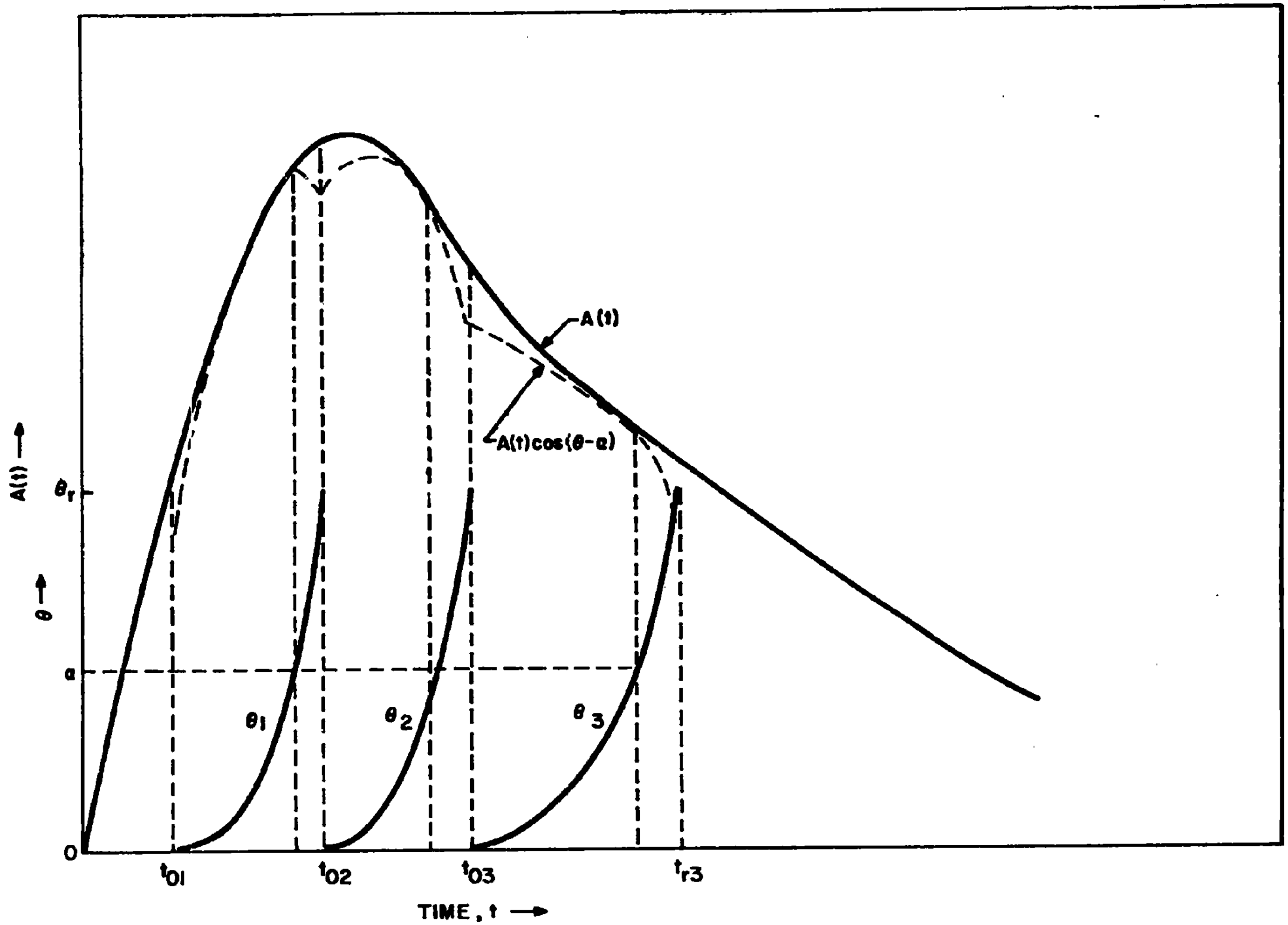

Figure 2. Modification of acceleration A(t) by cos (θ-α).

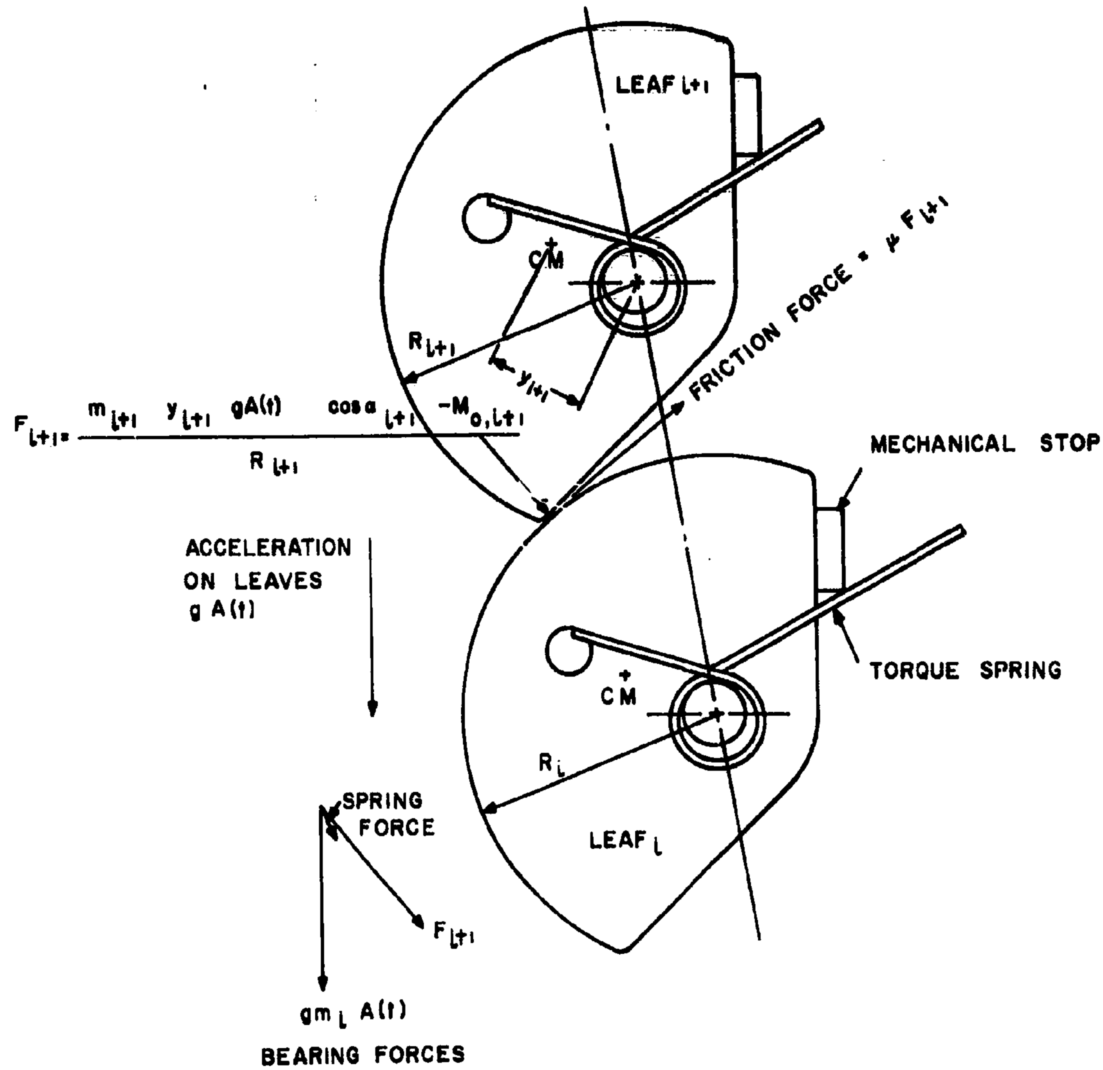

Figure 3. Functional forces on leaf.

where R is the leaf radius and μ_R is the coefficient of friction between leaf surfaces. The expression within parentheses is restricted to positive values, so that this torque is never negative. For the last leaf, this torque represents the pressure due to a rotor, release latch, or some such device. Because of lack of information as to what this element might be, it will be assumed for simplicity that its torque is of the same nature as that on the other leaves.

The friction torque resulting from the ith leaf pressing against its bearing is caused mostly by the force of the (i+1) leaf F_{i+1} (3) plus the force from the gun acceleration. Although these forces should properly be added vectorially (fig. 3), there is little error in adding them algebraically, with considerable simplification. The bearing friction torque is then

$$M_b = \mu_b\, r \left[m_i\, g\, A(t) + \frac{1}{R_{i+1}} \left(m_{i+1}\, y_{i+1}\, g\, A(t) \cos \alpha_{i+1} - M_{o,i+1} \right) \right] \quad (5)$$

r being the radius of the bearing in which rotates the shaft on which the leaf is mounted. Again, the expression within parentheses is restricted to positive values.

The small friction torque caused by the pressure of the helical spring against the leaf shaft (which should also be a small component of M_b) will be neglected in this analysis. Since little is known of the coefficients of friction and their variations under different conditions, there is no point in deriving every friction torque to a high precision. The friction torque expressions are only approximations, in effect, modifying the frictionless equations of motion.

If these torques are now substituted in the equation of motion (1) of leaf i, it becomes

$$I_i \frac{d^2 \theta_i}{dt^2} + M_{oi} + \lambda\theta_i + \mu_R \frac{R_i}{R_{i+1}} \left(m_{i+1}\, y_{i+1}\, g\, A(t) \cos \alpha_{i+1} - M_{o,\, i+1} \right)$$

$$+ \mu_b\, r \left[m_i g\, A(t) + \frac{1}{R_{i+1}} \left\{ m_{i+1}\, y_{i+1}\, g\, A(t) \cos \alpha_{i+1}\ \ M_{o,\ i+1} \right\} \right] = m_i y_i\, gA(t)$$

This equation can now be rearranged as follows:

$$I_i \frac{d^2\theta_i}{dt^2} + \lambda\theta_i = m_i y_i g\, A(t) \left[1 - \mu_b \frac{r}{y_i} - \frac{m_{i+1}\, y_{i+1}}{m_i\, y_i} \left(\frac{\mu_b r + \mu_R R_i}{R_{i+1}}\right) \cos \alpha_{i+1}\right]$$

$$- M_{oi}\left[1 - \frac{M_{o,\, i+1}}{M_{oi}} \left(\frac{\mu_b\, r + \mu_R\, R_i}{R_{i+1}}\right)\right] \qquad (6)$$

The equation of motion of each leaf is of this form.

2.2 Reduction of the Equation of Motion

Equation (6) can be simplified if certain reasonable assumptions are made. For reasons of safety, it is desirable in the manufacture of S&A devices to make the assembly as "foolproof" as possible. Therefore, the springs for all the leaves are usually made identical, and the leaves made of the same geometry, varying only in thickness (mass). Throughout this analysis it will be assumed that these requirements hold. Therefore, the subscripts will be dropped from each M_o, y, R, and α, because they will be the same for each leaf. Also, it will be assumed that the coefficients of friction μ_R and μ_b are the same, and their subscripts dropped. Then (6) reduces to

$$I_i \frac{d^2\theta_i}{dt^2} + \lambda\theta_i = m_i y\ g\ A(t) \left[1 - \mu\left\{\frac{r}{y} + \frac{m_{i+1}}{m_i} \left(1 + \frac{r}{R}\right) \cos \alpha\right\}\right]$$

$$- M_o \left[1 - \mu\left(1 + \frac{r}{R}\right)\right] \qquad (7)$$

If the expressions within brackets are each represented by a constant,

$$C_1 = 1 - \mu\left\{\frac{r}{y} + \frac{m_{i+1}}{m_i} \left(1 + \frac{r}{R}\right) \cos \alpha\right\} \qquad (8)$$

$$C_2 = 1 - \mu \left(1 + \frac{r}{R}\right) \qquad (9)$$

this simplified friction model is seen to be of the same form as the frictionless model.

$$I_i \frac{d^2\theta_i}{dt^2} + \lambda\theta_i = m_i\ ygC_1 A(t) - C_2 M_o \qquad (10)$$

For the frictionless case, C_1 and C_2 become unity. For a given geometry, ratio $\frac{m_{i+1}}{m_i}$, and constant value of the coefficient of friction, the equation of motion may be considered as having an acceleration and initial spring torque that are modified by the factors C_1 and C_2:

$$A' = C_1 A; \quad 0 \leq C_1 \leq 1$$

$$M_o' = C_2 M_o; \quad 0 \leq C_2 \leq 1$$

The equation of motion (10) is still not complete. It is necessary to account for the restraints on the leaves. All leaves are prevented from having a negative angle of rotation θ by suitable mechanical stops. In addition, leaves other than the first cannot move until the preceding leaves are out of the way. These restraints can be accounted for mathematically by including in the equation the step function:

$$\begin{aligned} h(t - t_o) &= 1 \text{ for } t - t_o > 0 \\ &= 0 \text{ for } t - t_o \leq 0 \end{aligned} \tag{11}$$

The corrected equation of motion then becomes:

$$\frac{d^2\theta_i}{dt^2} + \frac{\lambda}{I_i}\theta_i = h(t - t_{oi})\left[\frac{m_i yg}{I_i} C_1 A(t) - \frac{C_2 M_o}{I_i}\right] \tag{12}$$

This equation is applicable to all of the leaves. For the first leaf, t_o is the time at which the external applied acceleration has increased to the point where the applied torque equals the initial spring torque M_o; i.e., when the forcing function within the brackets goes positive. The other leaves are suddenly released later, at which times the applied torque is much larger than the spring torque. This is illustrated in figure 4, where t_{01}, t_{02}, t_{03} represent the times when each of the three leaves begins to rotate. The time when the first leaf moves (for a given applied acceleration) depends on the initial spring torque, but the other leaves start only when the preceding leaves in the sequence have moved out of the way. Their starting times are not a function of the constants of their own equation of motion, but depend on the solutions for the first leaves. Thus, the arming time of leaves other than the first one depends on the times that the first leaves take. In other

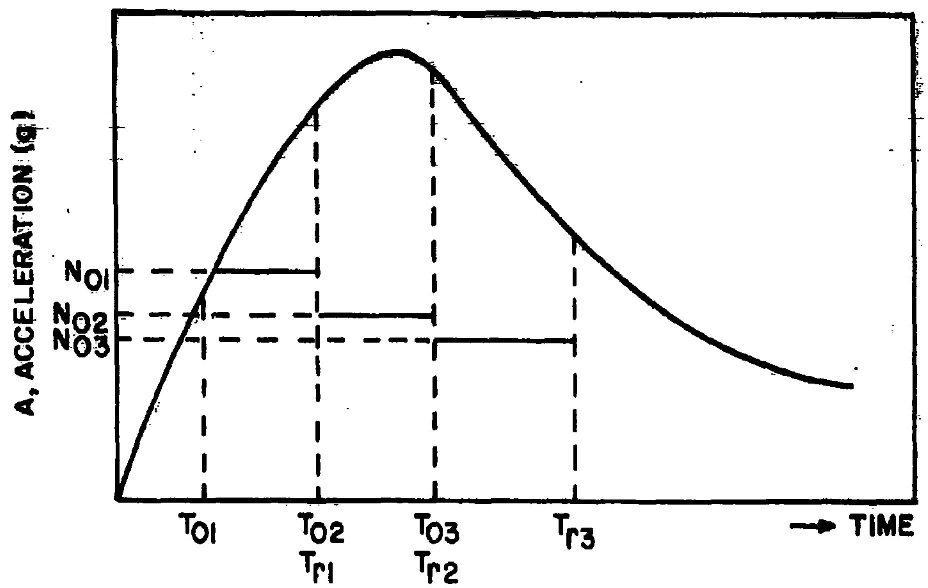

Figure 4. Initial starting times for each leaf rotation.

words, the arming time of a particular leaf depends upon where the release time occurs on the applied acceleration curve. The major analytical approach to this study relies on the determination of how the arming time varies as a function of the release time of a given leaf. This information, which has not been obtained before for representative gun accelerations, is a valuable aid to the design procedures for setback-leaf systems.

The initial spring torque M_o can be expressed in terms of equivalent g-level by letting (ref 1)

$$M_o = \lambda\theta_o = mygN_o \tag{13}$$

where N_o is now the g-level of applied acceleration to produce a torque equivalent to the initial spring torque. If this is substituted in equation (12), it becomes of the form:

$$\frac{d^2\theta}{dt^2} + \frac{\lambda}{I}\theta = h(t - t_o)\frac{myg}{I}\left[C_1A(t) - C_2N_o\right] \tag{14}$$

2.3 Method of Solution

This equation is recognized to be a second order linear differential equation of a simple undamped harmonic oscillator set into motion by a transient impressed forcing function. Because of this forcing function, or externally applied energy, the mass-spring system is nonconservative. However, it can be readily solved by standard techniques for a variety of acceleration functions of time. Solutions for some simple linear functions are derived in appendix B. Laplace transform techniques are used to solve differential equations throughout this analysis. These techniques are used because of their algebraic simplicity, automatically incorporating in the operational solution the initial or boundary conditions, and they are particularly adapted to handling equations containing step functions and delta functions. In all cases in this report, the initial conditions are that θ and $\frac{d\theta}{dt}$ are zero at t equals zero.

Although solutions are readily derived for θ, the expressions are so complicated that [except for the case of A(t) equal to a constant] analytical expressions cannot be obtained for other variables of more interest, particularly the arming time, or time for a leaf to rotate a given number of degrees. Therefore, analog computer solutions will be developed for these variables. The only solutions of interest are through the initial arming angle θ_r of leaf rotation. By the time θ reaches this value, the next leaf has been released, and subsequent performance is of no interest.

In the solutions of equation (14), λ/I is equal to the square of the natural frequency of oscillation of the mass-spring system without any forcing function. Therefore, λ/I can be replaced by β^2 in the equation of motion. In addition, if the moment of inertia I is replaced by its value mk^2, where k is the radius of gyration, the mass of the leaf is cancelled from the amplitude factor of the forcing function. However, θ is not independent of the mass, since it is a factor in the relationship between the spring torque and the acceleration as seen by equation 13, and in the term $\frac{\lambda\theta}{mk^2}$.

$$\frac{d^2\theta}{dt^2} + \frac{\lambda\theta}{mk^2} = h(t - t_o) \frac{yg}{k^2} \left[C_1 A(t) - C_2 N_o\right] \qquad (15)$$

It is assumed in this analysis that the leaves vary only in mass, the release angle θ_r, y, and k being appropriate constants. It is further assumed that the initial spring torque M_o is a constant of suitable value, with the relationship between M_o and N_o determined by equation 13 and the mass of a particular leaf.

Another variable will also be allowed, which is proportional to the spring constant λ. This variable is defined by first considering the relative change in the spring torque as θ rotates from its initial position to its arming position θ_r (fig. 5). The spring torque then increases from M_o to $M_o + \lambda\theta_r$, a change equal to $\lambda\theta_r$. The relative change in spring torque, which will be called z, is then

$$z = \frac{M_r - M_o}{M_o} = \frac{\lambda\, \theta_r}{M_o} \qquad (16)$$

M_o and θ_r are constants so that the variable z is proportional to the spring constant λ. (z is used as the variable instead of λ because it was found to be a useful dimensionless parameter in evaluating the drop safety.)

In summary, the variables in the analysis are z, the relative spring torque change, and the individual leaf masses m for various given gun acceleration-time curves. The problem then reduces to that of determining a method of selecting the three best masses for the leaves, which will result in an S&A device that will arm for the given applied acceleration-time function while requiring that the minimum velocity-to-arm be as large as possible. This minimum velocity-to-arm for a three-leaf device is defined as the sum of the minimum velocities-to-arm of the individual leaves. This problem is analyzed for representative acceleration-time functions of different shape, and for springs of different stiffness as given by z. The masses can vary from leaf to leaf, but the springs are the same for all three leaves. M_o, y, k, and θ_r have the same constant values throughout the analysis.

It is assumed in this analysis that the acceleration-time curve used is already the minimum that will be available with the lowest increment propellant. It is this applied acceleration that must arm the device. It is assumed that, if it arms for this minimum acceleration, it will arm for larger acceleration curves of the same shape.

The analysis is developed in detail in sections 4 through 7. There were two main results of this analysis. First, it is found from the procedure outlined above that there is very little to be gained by selecting a combination of leaves of different masses; i.e., by trying to choose the leaf mass to fit the particular segment of the acceleratiion function occurring at the time the leaf is rotating. For any combination of variable leaf masses designed to arm for the given applied acceleration and have the maximum drop-safety index, there is a set of equal-mass leaves that will also arm and have a drop-safety index that is no less than three or four percent below the index of the leaves of varying mass. Therefore,

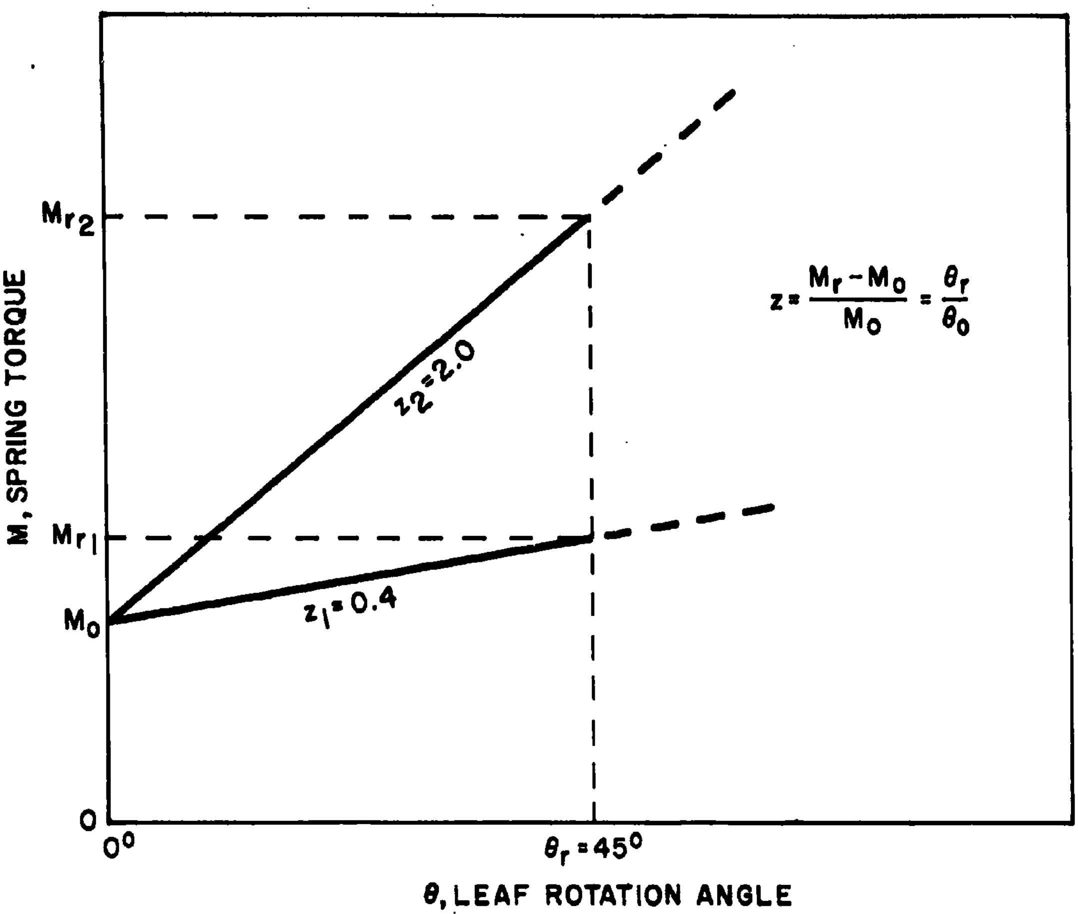

Figure 5. Spring torque rates.

unless there are other reasons for leaves of unequal mass, there is little to be gained by varying the mass from leaf to leaf. The design problem is greatly simplified by using leaves of the same mass.

The other major result of the analysis is the discovery that the leaf-spring system, or setback mechanism, can be designed from a flattened and "squared-up" acceleration-time curve of the same area. The area of the gun curve that is thus averaged is the upper part; i.e., above the level of the equivalent spring torque acceleration N_o (defined by eq 13 and illustrated by fig 6). The average acceleration above N_o can be obtained by finding the area by any of a number of methods and dividing it by the time interval. Then the optimum design is obtained by adjusting the physical parameters of the leaf-spring system until the setback mechanism just arms in the allowed time interval for a constant applied acceleration equal to this average.

The net acceleration required is obtained from the equation of motion of the leaf (eq 15) as shown below.

The solution to equation 15 for a constant applied acceleration A is shown in appendix C (eq C-17) to be

$$\theta = \frac{myg\ (C_1A - C_2N_o)}{\lambda}\ \left(1 - \cos\sqrt{\frac{\lambda}{mk^2}}\ t\right)$$

The leaf rotates to its arming angle θ_r in a time t_r.

$$\theta_r = \frac{myg}{\lambda}\ (C_1A - C_2N_o)\ \left(1 - \cos\sqrt{\frac{\lambda}{mk^2}}\ t_r\right) \qquad (17)$$

If λ is eliminated by use of equation (13) and equation (17) is rearranged, it becomes

$$C_1A = C_2N_o + \frac{\dfrac{\theta_r N_o}{\theta_o}}{1 - \cos\sqrt{\dfrac{yg}{k^2}\ \dfrac{N_o}{\theta_o}\ t_r^2}}$$

If C_1N_o is subtracted from both sides, the constant net acceleration, in excess of the "equivalent" spring torque acceleration, required to arm a leaf in a time t_r is found to be

$$C_1(A - N_o) = (C_2 - C_1)\ N_o + \frac{\dfrac{\theta_r N_o}{\theta_o}}{1 - \cos\sqrt{\dfrac{yg}{k^2}\ \dfrac{N_o}{\theta_o}\ t_r^2}} \qquad (18)$$

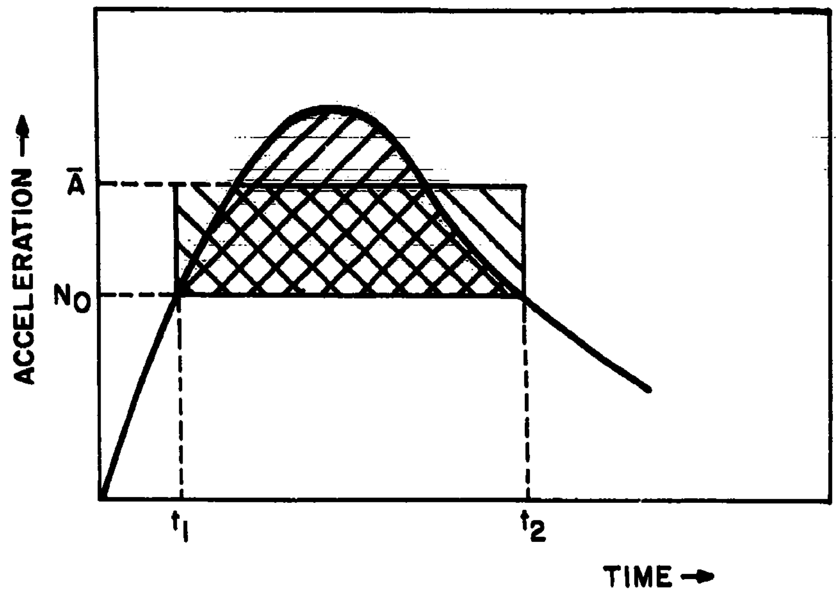

Figure 6. Average acceleration above N_o.

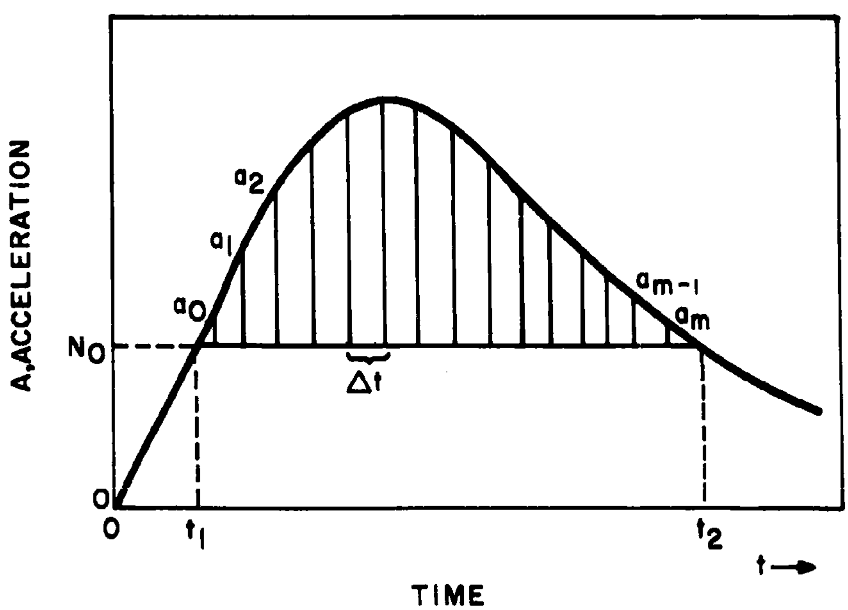

Figure 7. Calculation of area by trapezoidal rule.

The design technique that employs this formula is outlined in detail in the next section followed by an illustrative example.

3. DESIGN PROCEDURES

3.1 Simplified Formulas

This section outlines a procedure for designing a three-leaf setback mechanism having identical leaves and helical springs. The equations and procedures used are derived in later sections of this report. The method described is proposed for an optimum design where it is necessary that maximum safety be obtained from the setback mechanism against arming from accidental drop. At the same time, the device must arm in a reliable manner when fired at its lowest charge increment.

The design procedure, which is described in detail so that little mathematical background is required to follow it, consists of the following steps:

(1) The first requirement in designing the three-leaf mechanism is to obtain a graphical plot of the gun acceleration as a function of time. The acceleration should be in "g" or gravity units, and the time in milliseconds. This acceleration-time curve should be the minimum that might be available.

(a) From the gun curve, select an acceleration level N_o approximately half the peak value. Determine the time t_1 that the gun acceleration takes to rise to this value N_o, and the time t_2 when the gun acceleration drops again to N_o.

(b) Determine the area under the acceleration-time curve above the acceleration level N_o. This can be obtained by any of a number of standard techniques. Perhaps the simplest technique is to add evenly spaced (in time) values of the acceleration difference, and then to multiply by the time interval (fig. 7); this is the well-known trapezoidal rule for finding areas.

(c) Calculate the average value of acceleration above N_o by dividing the area by $t_2 - t_1$.

$$\overline{A - N_o} = \frac{\Delta t}{t_2 - t_1}\left[\frac{a_o}{2} + a_1 + a_2 + \ldots + \frac{a_n}{2}\right] \qquad (19)$$

where Δt is the time sub-interval between acceleration values, and $a_o, a_1, a_2, \ldots, a_n$ are the acceleration differences for each

point on the gun curve. Any triangular areas before a_o and beyond a_n should also be included before dividing by the term $(t_2 - t_1)$.

(d) Repeat steps a through c for other values of N_o. The range of values of N_o should be between one-quarter and three-quarters of the peak value of gun acceleration. Five to seven evenly spaced values of N_o are probably sufficient.

Once the first area is obtained, areas above other values of N_o can be found by a shortcut. The increase or decrease in area for a smaller, or larger value of N_o, respectively, closely approximates a trapezoidal area, which is easily calculated. This is illustrated in figure 8 for a smaller second value of N_o.

The incremental area is approximately equal to

$$\Delta(\text{Area}) = \frac{1}{2}(N_{01} - N_{02})\left[(t_{21} - t_{11}) + (t_{22} - t_{12})\right] \tag{20}$$

(e) Now plot both the average net acceleration $\overline{A - N_o}$ and the average arming time available for a single leaf,

$$t_r = \frac{t_2 - t_1}{3}, \tag{21}$$

as a function of N_o. (Let N_o be the abscissa.) Both curves will be approximately linear with negative slopes.

(2) The next step is to determine the leaf-spring parameters so that a leaf will just arm in a time t_r for the average net acceleration $\overline{A - N_o}$ obtained in step 1. The constant acceleration required to arm a leaf is given by equation (18). Since $\overline{A - N_o}$ and t_r are empirical functions of the chosen N_o, it is necessary to solve the equation (18) graphically.

It is assumed in this design procedure that the geometry of the setback mechanism has already been selected so that the following quantities are known:

R = radius of leaf

r = radius of leaf bearing shaft

k = radius of gyration of leaf

y = distance from axis of rotation to center of mass of leaf.

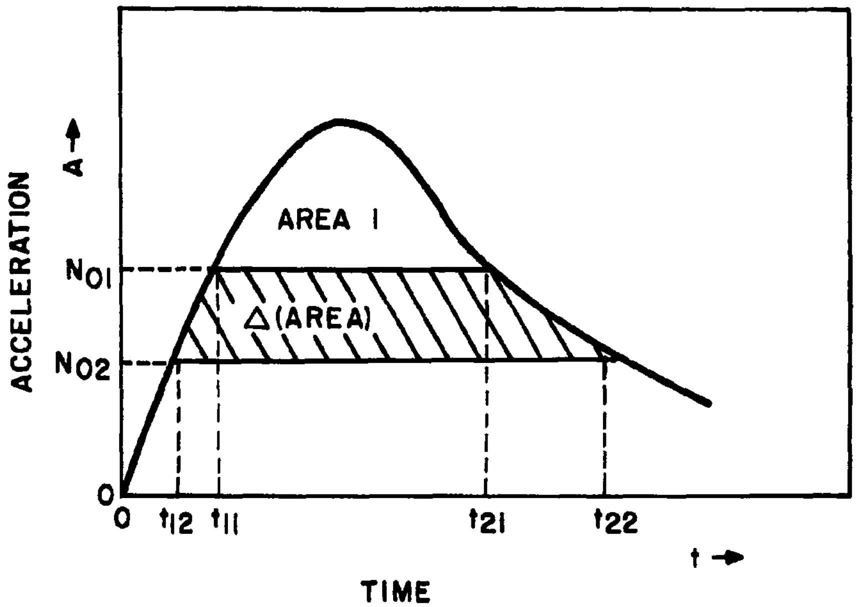

Figure 8. Trapezoidal area increment.

θ_o = initial windup angle of helical spring

θ_r = angle through which leaf must rotate to release next leaf, or to arm

D = mean diameter of coil

W = weight of leaf

μ = average coefficient of friction

α = initial angle between position of center-of-mass and direction perpendicular to acceleration

The graphical solution for N_o proceeds as follows:

(a) Calculate coefficients

$$C_1 = 1 - \mu\left(\frac{r}{y} + \left[1 + \frac{r}{R}\right] \cos \alpha\right) \tag{22}$$

$$C_2 = 1 - \mu \left(1 + \frac{r}{R}\right) \tag{23}$$

(b) Select a trial value of N_o approximately half of the gun acceleration peak. From the curves plotted in step 1(d), find the value of t_r.

Then calculate

$$(A - N_o) = \frac{C_2 - C_1}{C_1} N_o + \frac{\frac{N_o \theta_r}{C_1 \theta_o}}{1 - \cos \sqrt{\frac{yg}{k^2} \frac{N_o}{\theta_o}}\; t_r^2}, \tag{24}$$

and plot $(A - N_o)$ on the same graph paper used in step 1(d).

(c) If point $(A - N_o, N_o)$ is above the $\overline{A - N_o}$ versus N_o curve, select a smaller value of N_o, and repeat step 2(b). If the point is below the curve, a larger N_o should be used.

(d) Continue to repeat step (b) until enough points have been obtained to determine the value of N_o, at which $\overline{A - N_o}$ is equal to the calculated value of $A - N_o$; one-percent accuracy is sufficient.

(3) After determining optimum N_o, the next step is the design of the helical spring, its wire diameter, and number of coils. The diameter of wire depends on the allowable stress in the wire when the bending moment, or torque, is applied. The choice of torque at which the stress must be controlled is determined by the amount of torque on the wire when the leaf has rotated to its arming position θ_r. This torque is equal to

$$M_r = W\ y\ N_r = \lambda(\theta_o + \theta_r) \tag{25}$$

where N_r is the "equivalent acceleration" of the spring torque at the arming position and W is the weight of the leaf, mg.

(a) The minimum wire size is found as follows:

Calculate N_r from (fig. 9)

$$N_r = \frac{\theta_o + \theta_r}{\theta_o} \quad N_o \tag{26}$$

Find the torque at θ_r by

$$M_r = W\ y\ N_r = \frac{\theta_o + \theta_r}{\theta_o}\ M_o$$

The minimum wire size d is then obtained from

$$d^3 = \frac{32\ M_r}{\pi\ S_r} \tag{27}$$

where S_r is the maximum allowable stress at the arming position of a leaf (ref 5). Equation (27) is the expression for the stress in a cylindrical straight beam; while not strictly true for a coil of round wire, it is accurate enough for this purpose. The wire size selected is the smallest standard size equal to or greater than that obtained from equation (27).

(b) To determine the number of coils of helical wire, it is necessary to first find the desired spring constant λ.

$$\lambda = \frac{W\ y\ N_o}{\theta_o} = \frac{M_o}{\theta_o} \tag{28}$$

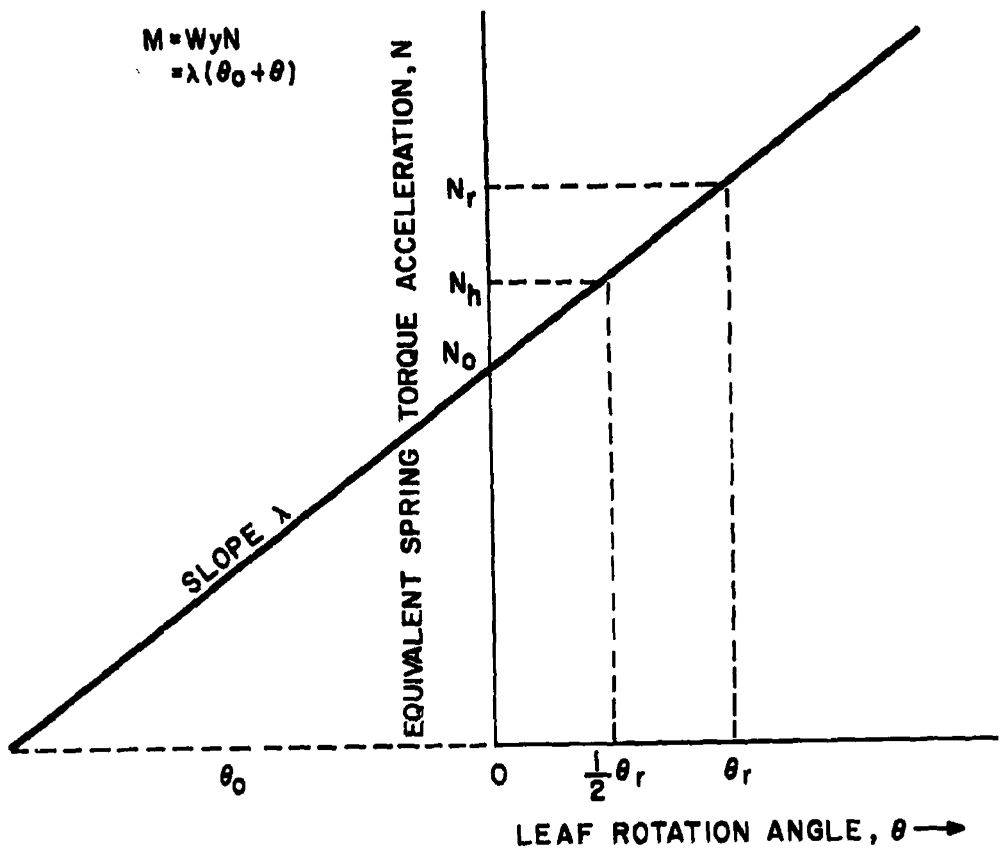

Figure 9. Relationship between spring angles and equivalent torque accelerations.

The standard formula for the spring constant of a coil is (ref 5)

$$\lambda = \frac{E\ d^4}{67.8\ nD} \tag{29}$$

where λ is the torque per radian, E is Young's modulus, D is the mean diameter of the helical coil, and n is the number of coils. Thus, the number of coils needed to provide the desired spring constant is

$$n = \frac{Ed^4}{67.8\ D\lambda} \tag{30}$$

n is usually not an integer because of the coil geometry and leaf arrangement. Thus, as shown by the example of figure 10, n might be

$$n = p - \frac{120^\circ}{360^\circ};\ p = 1,\ 2,\ 3,\ .\ .\ . \tag{31}$$

The smallest value of p is chosen that will make the value of n in equation (31) greater than that calculated from equation (30). With this number of coils, the height of the coil will be

$$h = (n + 1)\ d \tag{32}$$

(c) With d and n now determined, the actual value of the spring constant λ can be calculated from equation (29). This, in turn, requires an adjustment in either the weight of the leaf W, or the initial windup angle θ_o so that equation (28) is satisfied.

$$\theta_o = \frac{WyN_o}{\lambda} \quad \text{or} \quad W = \frac{\lambda\theta_o}{y\ N_o} \tag{33}$$

(No adjustment in N_o is needed since optimum N_o, obtained from eq 24, is insensitive to small changes in θ_o.)

The spring-and-leaf system parameters having now been obtained, the setback mechanism will consist of three identical leaves and springs operating in sequence.

As a check, the actual stress on a spring when its leaf rotates to the arming position is

$$S_r = \frac{32\ M_r}{\pi\ d^3} = \frac{10.2\ \lambda(\theta_o + \theta_r)}{d^3} \tag{34}$$

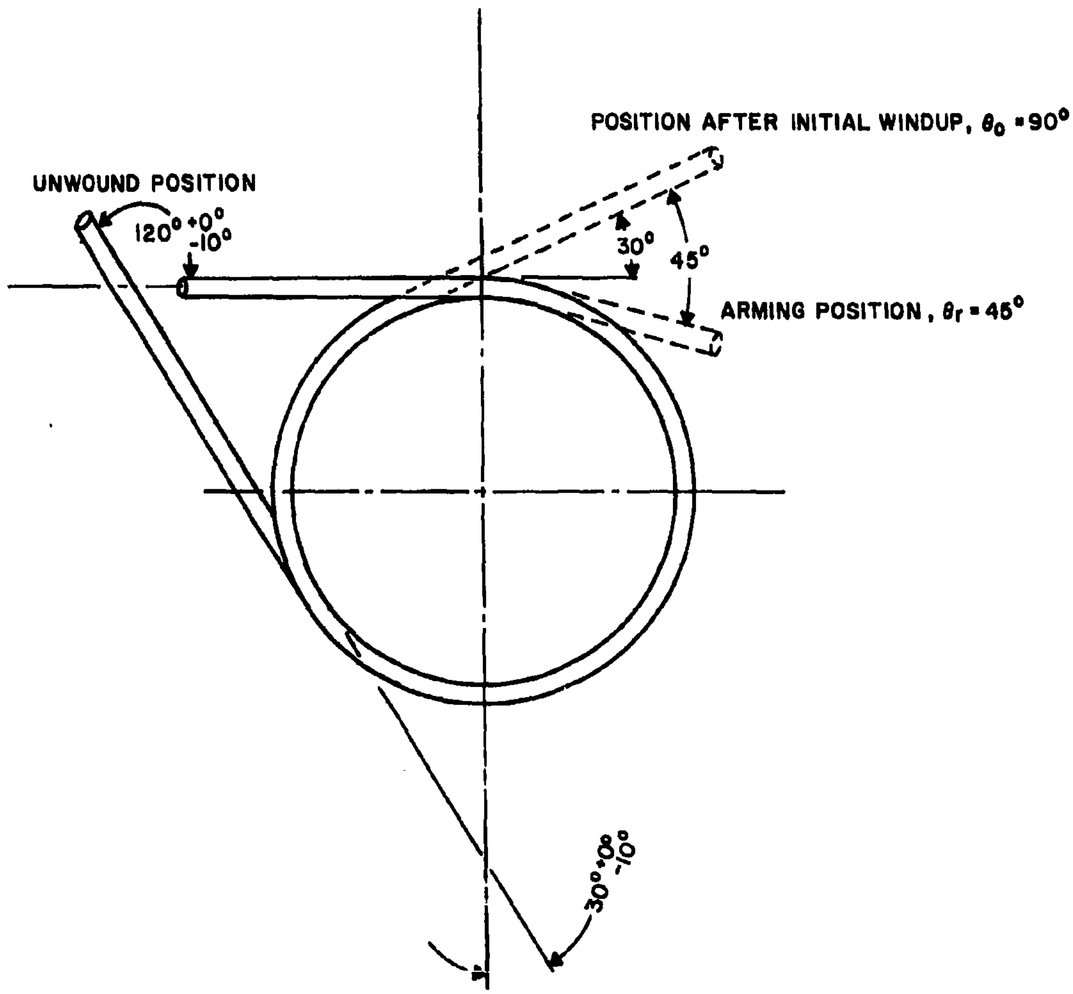

Figure 10. Helical wire coil.

(4) A setback mechanism having been designed to arm for the given gun curve and assumed parameters, the absolute minimum velocity change resulting from any accidental drop that will arm the device is shown in section 4 to be

$$V_\delta = 3\sqrt{2\,\frac{k^2}{y}\,\theta_r\,g\,N_h} \tag{35}$$

where N_h is the mean value of N halfway between N_o and N_r.

$$N_h = \frac{N_o + N_r}{2} = \frac{\theta_o + 1/2\,\theta_r}{\theta_o}\,N_o \tag{36}$$

V_δ is based on condition of no friction and with the velocity change occurring in the form of three equal delta functions properly spaced in time.

While V_δ is the absolute minimum velocity change that can arm the setback device, a more practical velocity change for specifying its expected drop safety is derived in section 4. This is the minimum velocity change for a constant applied acceleration, which continues until the first two leaves have armed and the third one has acquired just enough momentum to continue rotating to its arming position. This drop-safety velocity is given by

$$V_{ds} = p\,V_\delta \tag{37}$$

where p is a function of $z = \frac{\theta_r}{\theta_o}$ and is obtained from table I. Also given is the relative acceleration $\frac{A}{N_o}$, which requires the minimum velocity change. Any other value of acceleration will require a larger velocity change.

In general, it will be found that slightly larger values of V_δ and slightly smaller values of V_{ds} will be obtained for the larger values of z. However, practical spring manufacturing tolerances make it advisable to use large initial windup angles θ_o, and hence small z. The variation in end position of small coils cannot be kept less than about 10 deg without increased manufacturing costs. Therefore, the percentage error will be smaller for larger values of θ_o.

TABLE I. FACTORS FOR DETERMINING DROP-SAFETY VELOCITY

$z = \frac{\theta_r}{\theta_o}$	p	$\frac{A}{N_o}$
0.1	1.76	2.37
0.2	1.74	2.42
0.4	1.71	2.50
0.5	1.69	2.55
0.6	1.68	2.60
0.8	1.65	2.69
1.0	1.62	2.79
1.5	1.58	3.06
2.0	1.54	3.32

Since it is necessary that the mechanism arm reliably, the maximum value of θ_o allowed by this tolerance should be used in the design calculations. Other physical parameters such as k, y, and λ are not as likely to have a large manufacturing tolerance. However, if it is expected that any of these tolerances are not negligible, the maximum allowed values of k and λ and the minimum y should be used. This insures that the design value of N_o will be sufficiently small so that the device will arm despite the manufacturing variation in parameter values. Each of these changes in parameter values will cause a small reduction in the drop-safety velocity, but it is more important that the arming function reliably.

3.2 Example

(1) A setback-leaf mechanism will now be designed to arm the T28E6 shell fired at one increment. The acceleration-time curve is considered accurate to $\pm$ 10 percent. Therefore, since it is necessary that the device arm for the worst case, the acceleration curve used in the design must be for the lower limit (amplitude reduced by 10 percent) The resulting curve is shown in figure 11. The acceleration is seen to rise to an 800-g peak in a little more than 4 msec

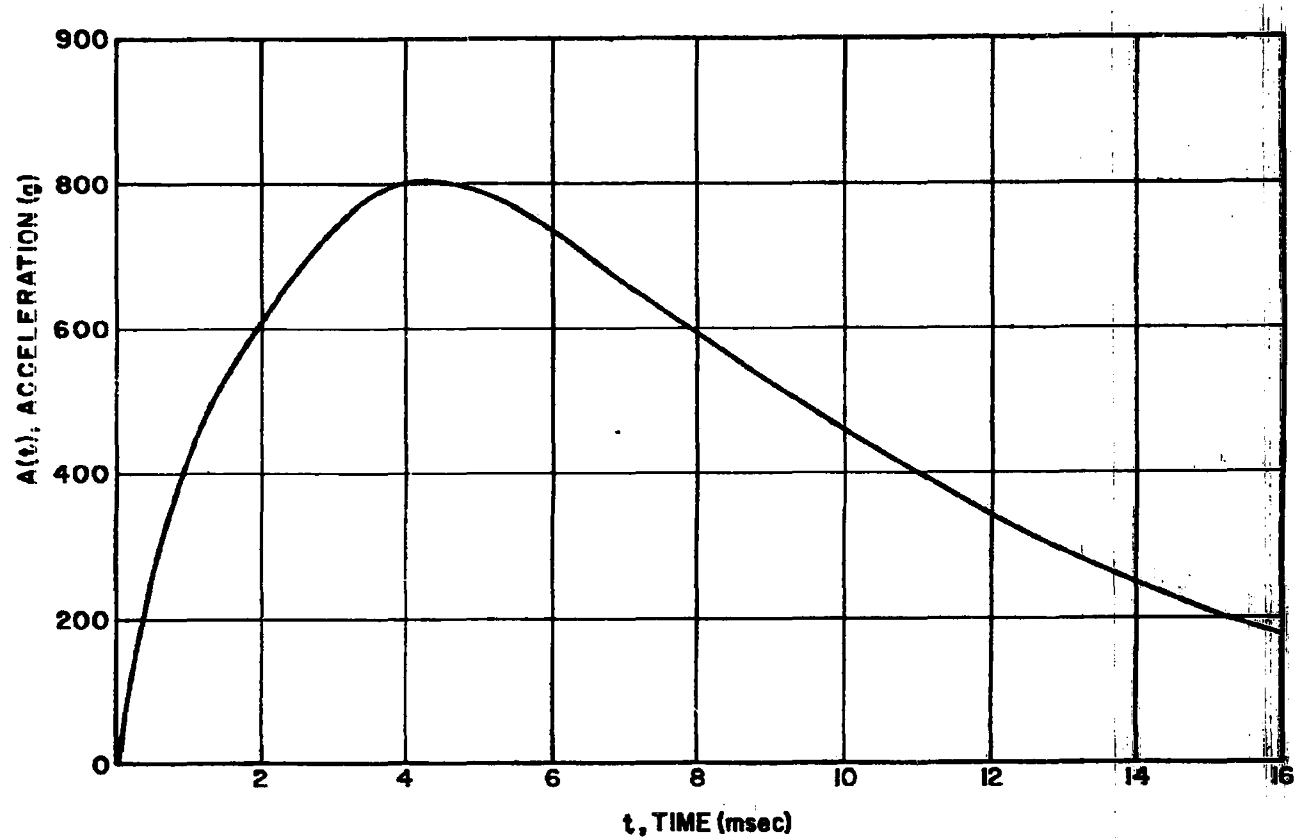

Figure 11. Acceleration of T28E6 mortar at one increment.

and then to drop more slowly than the rise time, the acceleration being below 200 g after 16 msec.

The procedure outlined earlier is now utilized.

(a) The first value chosen for N_o is 400 g. From the acceleration curve of figure 11, it is found that for $N_o = 400$ g,

$$t_1 = 0.9 \text{ msec}$$

$$t_2 = 11.0 \text{ msec}$$

$$t_r = \frac{11.0 - 0.9}{3} = \frac{10.1}{3} = 3.37 \text{ msec}$$

(b) The area between the acceleration curve and 400 g is found by adding the values of acceleration less 400 every 0.4 msec beginning at 1.2 msec. The small triangular areas before 1.2 msec and after 10.8 msec are added to the result to complete the area.

$$\text{Area} = 0.4\ [\frac{60}{2} + 135 + 205 + 255 + 307 + 352 + 385 + 400 + 400 + 392 + 380 + 360 + 340 + 308 + 275 + 250 + 220 + 192 + 164 + 137 + 100 + 83 + 55 + 32 + \frac{8}{2}\ ] + \frac{1}{2}\ [0.3 \times 60 + 0.2 \times 8] = 2318$$

(c) The average acceleration in excess of 400 g is

$$\overline{A - N_o} = \frac{2318}{t_2 - t_1} = \frac{2318}{11.0 - 0.9} = 230 \text{ g}$$

(d) Now let $N_o = 500$ g

Then $t_1 = 1.4$ msec

$$t_2 = 9.4 \text{ msec}$$

$$t_r = \frac{9.4 - 1.4}{3} = \frac{8.0}{3} = 2.67 \text{ msec}$$

The approximately trapezoidal area which is added algebraically to area above 400 g is

$$\Delta(\text{Area}) = \frac{1}{2}\,(400 - 500)\,[(11.0 - 0.9) + (9.4 - 1.4)]$$

$$= -\frac{100}{2}\,[10.1 + 8.0] = -905$$

The area above 500 g is thus

$$\text{Area} = 2318 - 905 = 1413$$

The average acceleration is then

$$\overline{A - N_o} = \frac{1413}{8.0} = 177 \text{ g}$$

Continuing this process for $N_o = 600$ g

$t_1 = 2.0$ msec

$t_2 = 7.9$ msec

$$t_r = \frac{7.9 - 2.0}{3} = 1.97 \text{ msec}$$

$$\text{Area} = 1413 - \frac{100}{2}\,[8.0 + 5.9] = 718$$

$$\overline{A - N_o} = \frac{718}{5.9} = 122 \text{ g}$$

For $N_o = 300$ g

$t_1 = 0.6$ msec

$t_2 = 12.8$ msec

$$t_r = \frac{12.8 - 0.6}{3} = 4.07 \text{ msec}$$

$$\text{Area} = 2318 + \frac{100}{2}\,[12.2 + 10.1] = 3433$$

$$\overline{A - N_o} = \frac{3433}{12.2} = 281 \text{ g}$$

Finally, for $N = 200$ g

$t_1 = 0.4$ msec

$t_2 = 15.3$ msec

$$t_r = \frac{15.3-0.4}{3} = 4.97 \text{ msec}$$

$$\text{Area} = 3433 + \frac{100}{2}[14.9 + 12.2] = 4788$$

$$\overline{A - N_o} = \frac{4788}{14.9} = 321 \text{ g}$$

(e) Now $\overline{A - N_o}$ and t_r are plotted for the five values of N_o and curves drawn through the points (figure 12). The curves are seen to be nearly linear. They show the average time and average net acceleration available from the firing of the shell to arm a leaf, as a function of the "equivalent acceleration" N_o of the initial spring torque.

(2) From the above acceleration curve data, a setback-leaf device will be designed, employing three identical leaf and spring combinations. Each set will just arm in the available time t_r for a constant applied acceleration A, so that $A - N_o$ is equal to the $\overline{A - N_o}$ available from the gun. From the geometry of the leaves, and an assumed coefficient of friction, are obtained the following necessary parameters for the design:

$R = 0.200$ in.

$r = 0.020$ in.

$D = 0.100$ in.

$k^2 = 0.0178$ in.^{2}

$y = 0.0645$ in.

$W = 0.0147$ ozf (ounces of force)

$\theta_o = 90$ deg (nominal) $= 1.57$ radians

$\theta_r = 45$ deg $= 0.785$ radians

$\alpha = 22\frac{1}{2}$ deg

$\mu = 0.2$

The units employed in this example will be inches for length, ounces for force and milliseconds for time. The acceleration due to gravity is, in these units: $g = 0.000386$ in./msec^2. Sample calculations for the radius of gyration k and the torque arm y are given in detail in Appendix A.

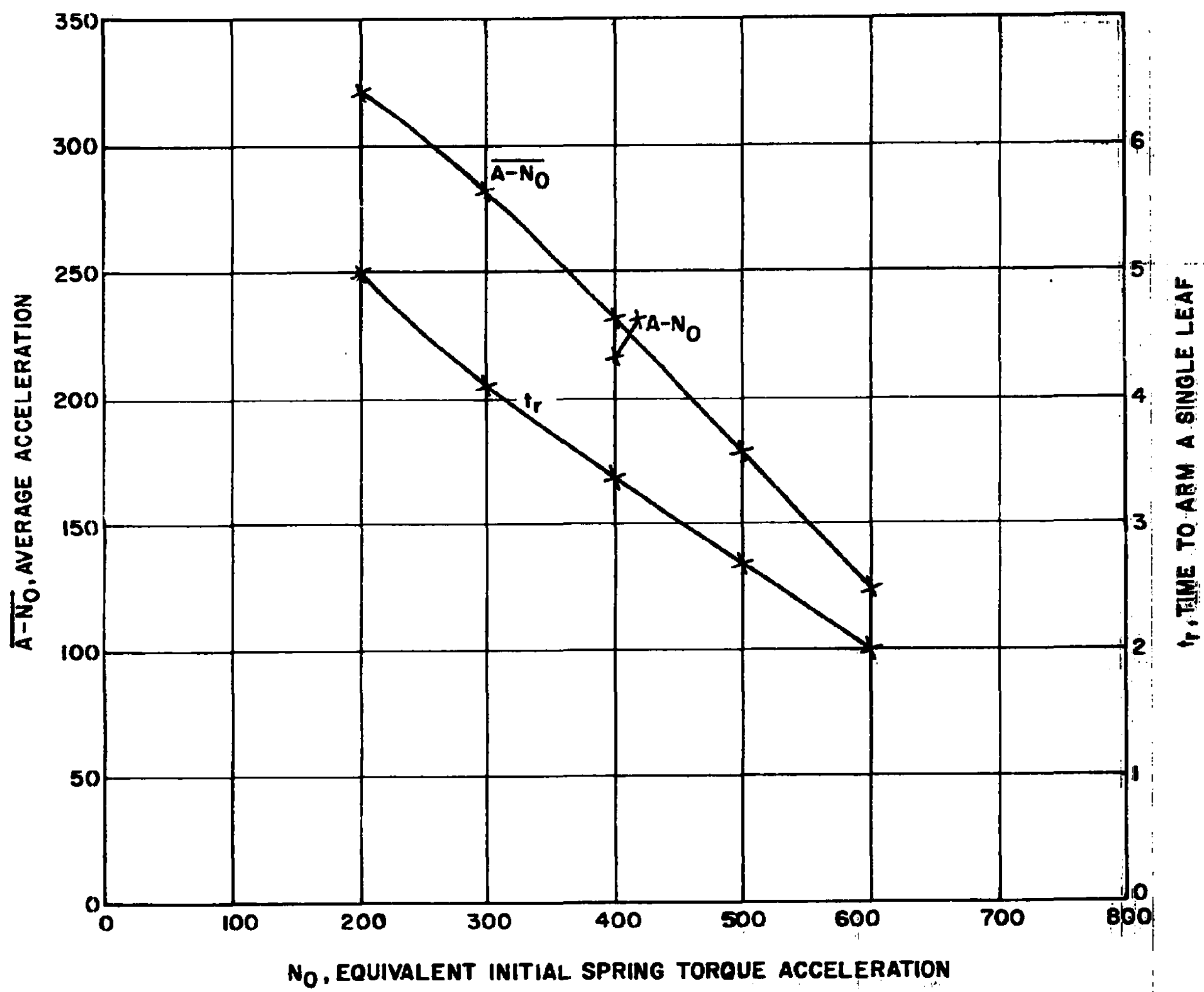

Figure 12. Average acceleration and arming time for each N_o.

(a) The coefficients C_1 and C_2 are now calculated.

$$C_1 = 1 - \mu\left(\frac{r}{y} + \left[1 + \frac{r}{R}\right]\cos\alpha\right) = 1 - 0.2\left(\frac{0.02}{0.0645} + \left[1 + \frac{0.02}{0.20}\right]\cos 22.5^\circ\right)$$

$$C_1 = 0.735$$

$$C_2 = 1 - \mu\left(1 + \frac{r}{R}\right) = 1 - 0.2\left(1 + \frac{0.02}{0.20}\right) = 0.780$$

(b) The optimum value of N_o will be obtained by successive trials. Let the first trial value of N_o be 400 g. From figure 12, the time available for arming is found to be

$$t_r = 3.37 \text{ msec}$$

The net acceleration required to arm the leaf in this time is

$$A - N_o = \frac{C_2 - C_1}{C_1} N_o + \frac{\dfrac{N_o \theta_r}{C_1 \theta_o}}{1 - \cos\sqrt{\dfrac{y\, g\, N_o t_r^2}{k^2\, \theta_o}}}$$

Since $\dfrac{y\, g}{k^2\, \theta_o} = \dfrac{0.0645 \times 0.000386}{0.0178 \times 1.57} = 0.000891,$

$$A - N_o = \frac{0.780 - 0.735}{0.735}\, 400 + \frac{\dfrac{400 \times 45^\circ}{0.735 \times 90^\circ}}{1 - \cos\sqrt{0.000891 \times 400 \times 3.37^2}}$$

$$A - N_o = 24 + \frac{272}{1 - \cos 2.01}$$

$$A - N_o = 215 \text{ g}$$

Thus, it is seen that for N_o = 400 g, it takes a net acceleration $A - N_o$ of only 215 g to arm in 3.37 msec, whereas there is available a firing acceleration of 230 g. Therefore, a larger N_o can be used. The point N_o = 400, $A - N_o$ = 215 is plotted on figure 12. It is seen

from this figure that $N_o = 420$ g may be the optimum value.

(c) Therefore, step (b) is repeated with $N_o = 420$ g for which $t_r = 3.23$ msec

$$A - N_o = \frac{0.780 - 0.735}{0.735} \times 420 + \frac{\frac{420 \times 45^0}{0.735 \times 90^0}}{1 - \cos \sqrt{0.000891 \times 420 \times 3.23^2}}$$

$$A - N_o = 26 + \frac{286}{1 - \cos 1.98}$$

$$A - N_o = 231 \text{ g}$$

Now, the net acceleration required, $A - N_o = 231$ g, is greater than the amount available, $\overline{A - N_o} = 220$ g. Therefore, the optimum value of N_o is between 400 and 420 g.

(d) The point $N_o = 420$ g, $A\text{-}N_o = 231$ g is plotted on figure 12, and a straight line is drawn between this point and the point obtained in step 2 (b). This $A - N_o$ line intersects the $\overline{A - N_o}$ line at about $N_o = 412$ g, $A - N_o = 225$ g. Thus, the largest value of N_o that can be used and still arm the three-leaf device is about 412 g. It is desirable to make N_o as large as possible to obtain maximum safety from an accidental drop.

(3) Now the helical spring opposing the rotation of the leaf is designed. It is desirable to keep the stress in the spring wire (usually music wire) below 150,000 psi. Therefore, the spring is designed to have a stress no greater than this amount when the coil is wound to its armed position θ_r. The coil at this time will be wound through a total angle equal to $\theta_o + \theta_r$.

(a) N_r, the "equivalent acceleration" at θ_r, is found from N_o.

$$N_r = \frac{\theta_o + \theta_r}{\theta_o} N_o = \frac{90^0 + 45^0}{90^0} \times 412 = 618 \text{ g}$$

The torque on the wire M_r at θ_r is then

$$M_r = Wy N_r = 0.0147 \times 0.0645 \times 618 = 0.586 \text{ in.-ozf}$$

The minimum wire diameter that can be used and still keep the stress below 150,000 psi when the torque, or bending moment is 0.586 in.-ozf is

$$d = \left(\frac{32 \times 0.586}{\pi \times 150{,}000 \times 16}\right)^{\frac{1}{3}} = 0.0136 \text{ in.}$$

Three standard wire sizes are 0.013, 0.014, and 0.016-in. diameters. Therefore, a wire diameter of 0.014 in. is selected so that the stress stays within the allowable limit of 150,000 psi.

(b) The desired spring constant λ for an equivalent acceleration N_o of 412 g when the spring has an initial windup angle θ_o of 90 deg is

$$\lambda = \frac{WyN_o}{\theta_o} = \frac{0.0147 \times 0.0645 \times 412}{1.57} = \frac{0.391}{1.57} = 0.249 \ \frac{\text{in-ozf}}{\text{rad}}$$

The number of coils n of 0.014-in. wire needed to provide this spring constant is

$$n = \frac{Ed^4}{67.8 \ D \ \lambda} = \frac{0.3 \times 10^8 \ (0.014)^4 \times 16}{67.8 \times 0.100 \times 0.249} = 10.92 \text{ coils}$$

where Young's modulus is 30,000,000 psi for steel wire. Because of the geometry of the coil leads, it is desirable that (fig. 10)

$$n = p - \frac{120^0}{360^0} = p - \frac{1}{3} \qquad p = 1, 2, 3, \ldots$$

Therefore, let p = 12, so that n = 11.67 coils. The height of the coil is then

$$h = (n + 1)d = 12.67 \times 0.014 = 0.177 \text{ in.}$$

(c) With d = 0.014 in. and n = 11.67 coils, the actual value of λ is

$$\lambda = \frac{Ed^4}{67.8 \ Dn} = \frac{0.3 \times 10^8 (1.4)^4 \times 10^{-8} \times 16}{67.8 \times 0.100 \times 11.67} = 0.233 \ \frac{\text{in-ozf}}{\text{rad}}$$

The actual windup angle θ_o for $\lambda = 0.233 \frac{\text{in-ozf}}{\text{rad}}$ must then be adjusted to a new value:

$$\theta_o = \frac{WyN_o}{\lambda} = \frac{0.0147 \times 0.0645 \times 412}{0.233} = 1.68 \text{ radians} = 96 \text{ deg}$$

to maintain an N_o value of 412 g. (An alternative adjustment could be made in W if it is desired to maintain θ_o equal to 90 deg; this would require only a small decrease in the thickness of the leaves.)

In summary, the spring will consist of 11.67 coils of 0.014-in. music wire with a height of 0.177 in. The spring constant is 0.233 $\frac{\text{in-ozf}}{\text{rad}}$ and the initial windup is 96 deg where the torque is 0.391 in-ozf, equivalent to an acceleration of 412 g.

The stress in the wire when the leaf rotates to its arming position θ_r will be

$$S_r = \frac{10.2\ \lambda\ (\theta_o + \theta_r)}{d^3} = \frac{10.2 \times 0.233\ (1.68 + 0.78)}{0.014^3}$$

$$= 2{,}130{,}000\ \frac{\text{ozf}}{\text{in}^2};\ \text{or, } S_r = 133{,}000 \text{ psi.}$$

This is well within the maximum allowance.

The setback mechanism consists of three of these identical leaves and springs, and is now designed to just complete the arming operation with little, if anything, to spare for the assumed minimum acceleration curve and constant coefficient of friction. It is the limit to which the design can be "stretched" with confidence that the device will perform its arming function. It is intended to obtain the maximum safety against arming from the sudden deceleration caused by an accidental drop.

The absolute minimum velocity change that must occur before the setback mechanism will arm depends on the value of the equivalent acceleration N_h when the torque on a leaf has rotated the leaf halfway to its armed position.

$$N_h = \frac{\theta_o + 0.5\ \theta_r}{\theta_o} N_o = \frac{96^\circ + 22\tfrac{1}{2}^\circ}{96^\circ} \times 412 = 508 \text{ g}$$

The absolute minimum velocity change required, which is that occurring in the form of three delta functions properly spaced in time with zero friction, is then

$$V_\delta = 3\sqrt{\frac{2k^2\ \theta_r g N_h}{y}} = 3\sqrt{\frac{2 \times 0.0178 \times 0.785 \times 0.000386 \times 508}{0.0645}}$$

$$V_\delta = 0.875\ \frac{\text{in.}}{\text{msec}} \text{ or } V_\delta = 72.9\ \frac{\text{ft}}{\text{sec}}$$

A more practical drop-safety velocity change would be a multiple of the above figure, obtained from table I for $\frac{\theta_r}{\theta_o} = \frac{45^\circ}{96^\circ} = 0.47.$

The minimum velocity change for a constant acceleration continuing until the third leaf has acquired enough momentum to reach its armed position is

$$V_p = pV_\delta = 1.70 \times 72.9 = 124\ \frac{\text{ft}}{\text{sec}}$$

if the constant acceleration is 2.54 N_o, or 1046 g.

If this velocity change is higher than necessary for reasonable safety precautions, the design can be relaxed somewhat so that the mechanism is more certain of arming, i.e., it will arm at a lower acceleration. This is done by increasing the thickness of the leaf, decreasing the spring constant λ, or decreasing the initial wind-up angle θ_o. The minimum velocity would, of course, have to be recalculated to adjust for changes in p or N_h.

4. DROP-SAFETY INDEX

The design procedure described is based on the results of an analysis of the operation of setback-leaf mechanisms, which is discussed in the remaining sections of this report. The method of approach to this problem was outlined in section 2. An optimum design is one that arms reliably when the projectile is fired at some specified velocity, but has maximum safety against arming from velocity changes caused by accidental handling drop-impacts. The design should be such that accidental acceleration operate only some of the leaves, but not all. The finite time that is required to arm each of the sequentially operating leaves insures that the device will not arm unless the accidental acceleration exists for some minimum length of time. This means that there must be some minimum velocity change (time integral of acceleration) to arm all the leaves regardless of the nature of the acceleration function.

Hausner thoroughly discussed the problem of safety; his results and conclusions are applied here. As a figure of merit for the optimum design, he proposed the absolute minimum velocity change required to arm all the leaves; the higher this figure becomes, the safer the design. Of course, the design must also be such that the mechanism arms during firing. The mathematical model used by Hausner assumed (for simplicity) that the resulting spring torque as the leaf rotated was constant, rather than increasing linearly. This resulted in smaller minimum velocity changes. Hausner's derivations are given in appendix C, together with the derivations for the case of linearly increasing spring torque (positive λ).

All these derivations related to the safety of a design assume that there is zero friction. Because friction consumes energy, there is less velocity change required where the friction is zero. Since it is always possible that, under some conditions of accidental drop, the friction may be very small, it is advisable to calculate velocity changes with friction assumed equal to zero.

To obtain the minimum velocity change to arm a three-leaf device, it is useful to first derive the velocity changes for two different applied accelerations required to arm a single leaf. The first acceleration for which the velocity change is calculated is that of a constant acceleration A, suddenly applied and lasting until a leaf arms. This velocity change is given by equation (C-5) for the case of a constant opposing spring force ($\lambda = 0$) as

$$V = \left(\frac{2gI\ \theta_r\ A^2}{my(A - N_o)}\right)^{\frac{1}{2}} \tag{38}$$

In addition, the velocity change is obtained for the case of a constant acceleration lasting only until the leaf has acquired sufficient momentum to rotate to its armed position. This velocity change is given by equation (C-15), again for the case of a constant opposing force.

$$V = \left(\frac{2gI\ \theta_r\ N_oA}{my(A - N_o)}\right)^{\frac{1}{2}} \tag{39}$$

These were the results obtained by Hausner. The velocity changes for the same two accelerations as above are derived in appendix C for positive λ and are given respectively by equations (C-22) and (C-28):

$$V = Ag \sqrt{\frac{I}{\lambda}} \cos^{-1} \left[1 - \frac{\lambda\theta_r}{myg(A - N_o)}\right] \quad (40)$$

$$V = Ag \sqrt{\frac{I}{\lambda}} \cos^{-1} \left[\frac{1 + \left(\frac{A - N_o}{A}\right)^2 - \left(\frac{mygN_o + \lambda\theta_r}{mygA}\right)^2}{2\left(\frac{A - N_o}{A}\right)}\right] \quad (41)$$

Equations (38) and (39) are special cases of equations (40) and (41) where λ equals zero.

These equations can be plotted in dimensionless units. To do this, however, it is first necessary to express the equations in somewhat different form by employing equations (12), (13), and (16). Then the following equivalent expressions are obtained:

$$\frac{\lambda \theta_r}{myg(A - N_o)} = \frac{\frac{\lambda\theta_r}{M_o}}{\frac{A}{N_o} - 1} = \frac{z}{\frac{A}{N_o} - 1} \quad (42)$$

$$Ag \sqrt{\frac{I}{\lambda}} = Ag\left(\frac{\frac{I\theta_r}{mygN_o}}{\frac{\lambda\theta_r}{M_o}}\right)^{\frac{1}{2}} = \left(\frac{gI\theta_r N_o}{my}\right)^{\frac{1}{2}} \frac{\frac{A}{N_o}}{\sqrt{z}} \quad (43)$$

$$\frac{mygN_o + \lambda\theta_r}{mygA} = \frac{1 + z}{\frac{A}{N_o}} \quad (44)$$

When these expressions are substituted in (40) and (41), they become

$$V = \left(\frac{I\theta_r gN_o}{my}\right)^{\frac{1}{2}} \frac{\frac{A}{N_o}}{\sqrt{z}} \cos^{-1} \left[1 - \frac{z}{\frac{A}{N_o} - 1}\right] \quad (45)$$

and

$$V = \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} \frac{\frac{A}{N_o}}{\sqrt{z}} \cos^{-1}\left[\frac{1 + \left(1 - \frac{N_o}{A}\right)^2 - \frac{(1+z)^2}{A^2/N_o^2}}{2\left(1 - \frac{N_o}{A}\right)}\right]$$

$$= \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} \frac{\frac{A}{N_o}}{\sqrt{z}} \cos^{-1}\left[\frac{\frac{A^2}{N_o^2} + \left(\frac{A}{N_o} - 1\right)^2 - (1+z)^2}{2\,\frac{A}{N_o}\left(\frac{A}{N_o} - 1\right)}\right]$$

or

$$V = \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} \frac{\frac{A}{N_o}}{\sqrt{z}} \cos^{-1}\left[1 - \frac{z(2+z)}{2\,\frac{A}{N_o}\left(\frac{A}{N_o} - 1\right)}\right] \tag{46}$$

Figure 13 is a plot of equations (45) and (46) where the abscissa is $\frac{A}{N_o}$ and the ordinate is

$$\frac{V}{\left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}}}$$

and the parameter for the different curves is the spring rate $z = 0.1$, 0.5, 1.0, 1.5, and 2.0.

Equations (38) and (39) are the special case of $z = 0$ after they are written in the same dimensionless units:

$$V = \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} \left(\frac{2\,\frac{A^2}{N_o^2}}{\frac{A}{N_o} - 1}\right)^{\frac{1}{2}} \quad ; \ z = 0 \tag{47}$$

and

$$V = \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} \left(\frac{2\,\frac{A}{N_o}}{\frac{A}{N_o} - 1}\right)^{\frac{1}{2}} \quad ; \ z = 0 \tag{48}$$

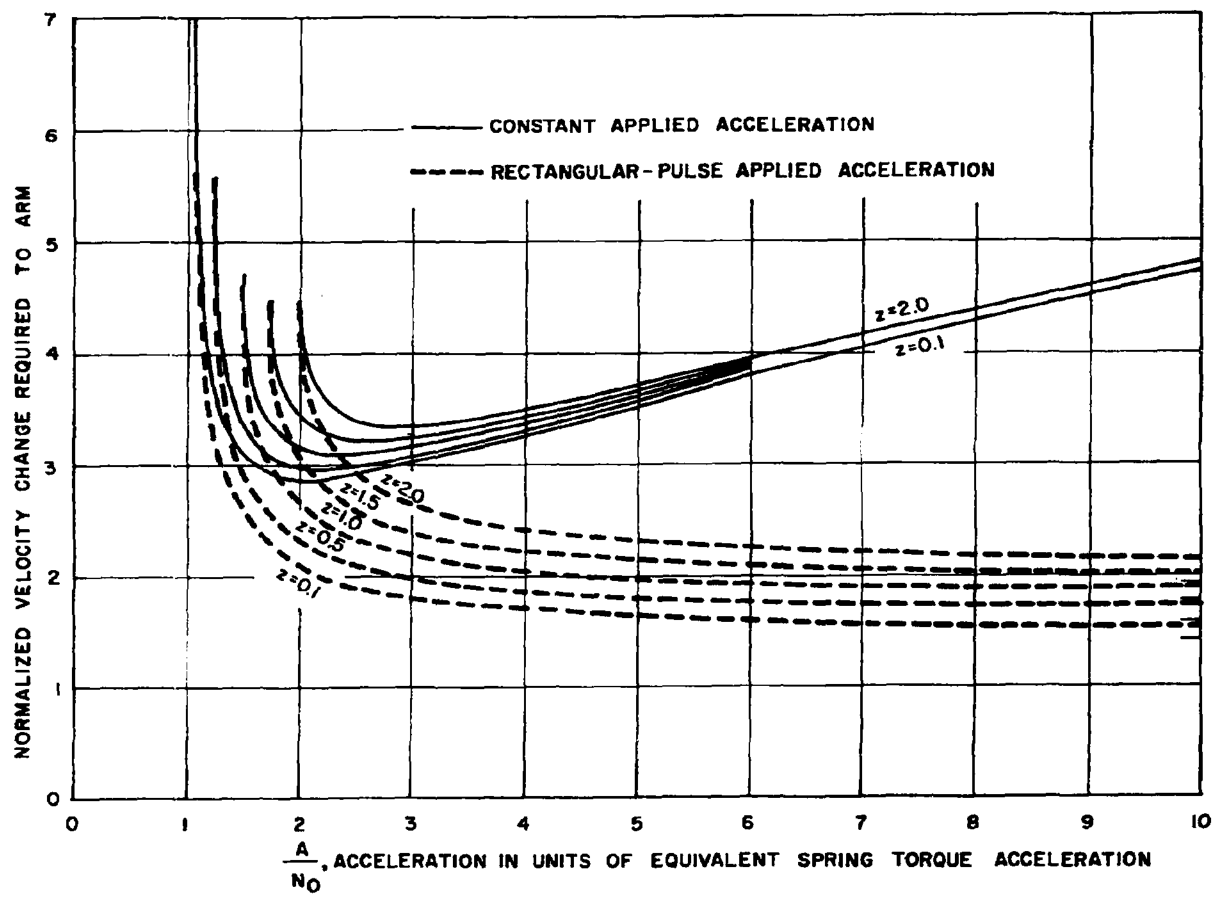

Figure 13. Velocity change required to arm a leaf.

Hausner's equations derived for $z = 0$ contain the factor $\cos\alpha$ in the denominators of (47) and (48) as he let $A(t)\cos\theta$ in (8) be $A\cos\alpha$; whereas, in this derivation it is represented by A only, the $\cos\alpha$ being absorbed into A. If it is desired to include $\cos\alpha$ in these equations, it can be done by replacing V in equations (45) to (48) by $V\cos\alpha$.

The solid curves of figure 13 represent equation (45) for the case of the constant acceleration continuing until the leaf rotates to θ_r, while the dashed curves are for (46)--the case of the rectangular pulse. These curves present the velocity change required to arm the leaf as a function of the acceleration A in units of N_o, the acceleration required to just overcome the opposing spring force and start the leaf rotating. It is seen from this figure that the leaf will not arm unless $\frac{A}{N_o}$ is greater than a particular value which is a function of z. This relationship between the minimum $\frac{A}{N_o}$ and z can be obtained from the requirements in equations (46) and (47) that the absolute value of the terms within the square brackets be no greater than unity. After suitable algebra, both (46) and (47) yield the same required inequality for the leaf to arm:

$$\frac{A}{N_o} > 1 + \frac{z}{2} \tag{49}$$

Since

$$z = \frac{\lambda\theta_r}{M_o} = \frac{\lambda\theta_r}{\lambda\theta_o} = \frac{\theta_r}{\theta_o},$$

it is necessary that

$$\frac{A}{N_o} > \frac{\theta_o + \frac{1}{2}\theta_r}{\theta_o}$$

or that, by (36),

$$A > N_h$$

Thus, it is seen that if a constant acceleration is suddenly applied to a leaf, (as in centrifuge testing), it will not arm unless the acceleration is greater than the equivalent acceleration N_h of the spring torque when the leaf has rotated halfway to its arming position.

The absolute minimum velocity to arm with the rectangular pulse is needed for large values of acceleration and, in fact, is least for accelerations infinitely large. If $\frac{A}{N_o}$ is allowed to approach infinity in (46) to obtain an expression for the value of this minimum velocity, the equation becomes indeterminate. An attempt to employ L'Hospital's rule results in laborious algebra; therefore, the expression was obtained using a series expansion for small values of the arc cosine (ref 6):

$$\cos^{-1} x = [2(1 - x) + \frac{1}{3}(1 - x)^2 + \frac{4}{45}(1 - x)^3 + \ldots]^{\frac{1}{2}}$$

where

$$1 - x = 1 - 1 + \frac{z(2 + z)}{2\frac{A}{N_o}\left(\frac{A}{N_o} - 1\right)} = \frac{z(2 + z)}{2\frac{A}{N_o}\left(\frac{A}{N_o} - 1\right)}$$

Thus,

$$V_{min} = \lim_{\frac{A}{N_o} \to \infty} \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} \frac{\frac{A}{N_o}}{\sqrt{z}} \left[\frac{z(2+z)}{\frac{A}{N_o}\left(\frac{A}{N_o} - 1\right)} + \frac{1}{3}\left(\frac{z(2+z)}{2\frac{A}{N_o}\left(\frac{A}{N_o} - 1\right)}\right)^2 + \ldots\right]^{\frac{1}{2}}$$

$$= \lim_{\frac{A}{N_o} \to \infty} \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} \left[\frac{(2 + z)}{\left(1 - \frac{1}{A/N_o}\right)} + \frac{1}{3}\left(\frac{\sqrt{z}\,(2+z)}{2\left(\frac{A}{N_o} - 1\right)}\right)^2 + \ldots\right]^{\frac{1}{2}}$$

or

$$V_{min} = \left(\frac{I\theta_r g N_o}{my}\right)^{\frac{1}{2}} (z + 2)^{\frac{1}{2}} \tag{50}$$

When I is replaced by mk^2 and equation (13) is substituted, this expression for V_{min} reduces to

$$V_{min} = \frac{k}{y}\sqrt{\theta_r M_o}\sqrt{\frac{2 + z}{m}} \tag{51}$$

where k, y, θ_r, and M_o are all constants as specified in section 2. This minimum velocity is the absolute minimum velocity that can arm the leaf. This is the velocity change that is experienced when a delta function acceleration is applied initially to the leaf. It is physically an impossibility, but it is the limiting case of a larger and larger rectangular pulse occurring for a shorter and shorter time, an instantaneous velocity change. Equation (50) is seen to agree with the solution (C-34) obtained independently for the case of an applied delta function. The time that it takes the leaf to arm is also a minimum for an applied delta function acceleration. This minimum time is given by equation (C-33):

$$t_{min} = \sqrt{\frac{I}{\lambda}} \tan^{-1} \sqrt{\frac{\lambda}{I}} \frac{V}{gN_o} \qquad (52)$$

If the numerator and denominator inside each square root are multiplied by $\frac{\theta_r}{M_o}$, and N_o eliminated by (13), the expression becomes

$$t_{min} = \left(\frac{mk^2\theta_r}{zM_o}\right)^{\frac{1}{2}} \tan^{-1} \frac{ygV}{k} \left(\frac{mz}{\theta_r M_o}\right)^{\frac{1}{2}} \qquad (53)$$

While the absolute minimum velocity change required to arm the leaf is that given by (51) (which is indicated in figure 13 as a limit of the curves by a horizontal line index at the right of the graph), it should be noted that the velocity changes required to arm much lower acceleration values of rectangular pulses are only a few percent larger than those required for delta functions. The delta function serves as a lower limit, but other rectangular pulses require only slightly larger velocity changes.

Now that the velocity changes needed to arm a single leaf have been derived for these acceleration functions, it is possible to derive velocity changes for arming the entire multiple-leaf setback system. The absolute minimum velocity change to arm three leaves is considered to be the sum of the absolute minimum velocities to arm each of the individual leaves, or

$$V_\delta = \sum_{i=1}^{3} V_{i\ min} \qquad (54)$$

for a three-leaf device, where V_δ is the total velocity change required with three delta functions. When equation (51) is substituted in (54), it becomes

$$V_\delta = \sum_{i=1}^{3} \sqrt{\frac{k^2 \theta_r M_o (2 + z)}{y^2 m_i}} \tag{55}$$

For the analysis of this report, the leaves have the same geometry, varying only in thickness (and therefore mass), and the springs are identical. If the mass is expressed as:

$$m_i = m_{ri} m_o \tag{56}$$

where m_o is some arbitrary mass and m_r is a dimensionless decimal multiplier, the minimum velocity change may be put in the form

$$V_\delta = \sqrt{\frac{k^2 \theta_r M_o}{y^2 m_o}}\; Z \tag{57}$$

where

$$Z = \sum_{i=1}^{3} \sqrt{\frac{2 + z}{m_{ri}}} \; . \tag{58}$$

Since k^2, θ_r, M_o, y, and m_o will be kept constant, the minimum velocity change is proportional to Z, which will hereafter be called the drop-safety index. By choosing a design that maximizes Z while successfully arming for the given gun acceleration, the absolute minimum velocity change is also maximized and the design is considered optimum. Section 5 and 6 present a method of selecting values of m_{r1}, m_{r2}, and m_{r3} for different values of z to obtain maximum values of Z and still arm when fired. The m_{ri} are functions of z, larger values of z requiring larger m_{ri} to arm, so that Z is not necessarily increased by increasing z.

Since V_δ is the absolute minimum velocity change required, any practical value of drop velocity change for which the setback-leaf

device would be expected to be safe would be larger than equation (57). About the only way three narrow pulse accelerations (step changes in velocity) could be approached would be in the highly improbable case of a drop through three successive metal plates properly spaced to allow each leaf time to arm. An approximately constant acceleration would appear to be the most probable type encountered.

Therefore, as a measure of the drop safety, consider the minimum velocity change required to arm for a constant applied acceleration, continuing until the first two leaves have rotated to their release position and the third has acquired sufficient momentum to continue the necessary rotation to complete the arming function. The velocity change required for any applied acceleration for each of the first two leaves is given by equation (45), and for the third leaf, by equation (46). When these velocity changes are summed and the result normalized with respect to V_δ, the following expression is obtained.

$$\frac{V}{V_\delta} = \frac{\frac{A}{N_o}}{3\sqrt{z(2+z)}} \left\{ 2\cos^{-1}\left[1 - \frac{z}{\frac{A}{N_o} - 1}\right] + \cos^{-1}\left[1 - \frac{z(2+z)}{2\frac{A}{N_o}\left(\frac{A}{N_o} - 1\right)}\right] \right\} \quad (59)$$

The minimum values (p) of this ratio as a function of z are obtained graphically (not shown) and given in table I. Then the minimum drop-safety velocity is given by

$$V_{ds} = p\, V_\delta$$

where p is obtained from table I, and V_δ, from equation (57). This should be a good practical drop-safety velocity for use in specifying the safety of the setback-leaf mechanism. As it is derived assuming zero friction, it is a conservative figure.

5. ARMING TIME

Now that a quantitative method has been defined for evaluating the relative safety of a particular design of a multiple-leaf setback mechanism, it is necessary to examine the other major factor in the choice of a design--the arming time of the device for a given gun acceleration. A method is needed for readily determining whether or not a particular design will successfully arm. The total arming time can be obtained if it is known how long it takes each leaf to arm. If the first leaves take so long that there is not sufficient gun acceleration left to arm the last leaf, a change must be made in the design. For all applied accelerations that are not constant, the time that it takes a leaf to rotate to any given angle will be a

function of the time at which it is released. The first leaf starts its motion when the applied acceleration rises to a value equal to the initial spring torque's "equivalent acceleration" N_o. The other leaves start their motion as soon as the preceeding leaves arm. Thus, they can start at any time on the acceleration curve.

Graphical curves will be developed in this section presenting the arming time versus the release time for various leaf masses. These curves will be obtained for two different gun acceleration-time curves, in order to determine the effect of the shape of the acceleration curve. One acceleration will be similar to that of the T28E6 mortar, whereas the other will have a faster rise to its peak value. In addition, two very different values of z will be used to investigate whether or not the relative change in spring torque of a leaf between its initial position and its arming position affects the performance. Friction will be assumed equal to zero although the method would be the same for constant non-zero friction.

It is shown in appendix E that gun acceleration curves can be represented by a function of the form

$$A(t) = A_o(e^{-at} - e^{-bt}); \; b > a$$

where the shape of the curve is varied by changing the ratio $\eta = \frac{b}{a}$.

The larger η is, the steeper is the rise. If this function is sustituted for A(t) in (14), the equation of motion for the (i)th leaf becomes

$$\frac{d^2\theta_i}{dt^2} + \frac{\lambda\theta_i}{I_i} = h(t - t_{oi}) \frac{m_i y\, g}{I_i} \left[C_1 A_o(e^{-at}\; e^{-bt}) - C_2 N_{oi}\right] \qquad (60)$$

The starting time for rotation of the first leaf t_{01} is determined by the time at which the term within the brackets becomes positive,

$$C_1 A_o\,(e^{-at_{o1}} - e^{-bt_{o1}}) = C_2 N_{o1} \qquad (61)$$

while the starting times of the second and third leaves are the times when the first and second leaves, respectively, reach their arming angles.

$$t_{o2} = t_{r1}$$

$$t_{o3} = t_{r2}$$

The initial spring torque and moment of inertia of each leaf are given by

$$M_o = m_i y g N_{oi}$$

$$I_i = m_i k^2$$

where M_o, λ, y, and k are the same for each leaf.

To simplify the algebra, let

$$\beta_i^2 = \frac{\lambda}{I_i} = \frac{M_o z}{m_i k^2 \theta_r} \tag{62}$$

$$P = \frac{C_1 m_i y g A_o}{I_i} = \frac{C_1 y g A_o}{k^2} \tag{63}$$

$$Q_i = \frac{C_2 m_i y g N_{oi}}{I_i} = \frac{C_2 M_o}{m_i k^2} \tag{64}$$

Equation (60) then becomes

$$\frac{d^2\theta_i}{dt^2} + \beta_i^2 \theta_i = h(t - t_{oi}) \; [P(e^{-at} - e^{-bt}) - Q_i] \tag{65}$$

The general solution to this equation is given by (D-3) (dropping the running index i),

$$\theta = h(t - t_o) \frac{P}{\beta^2} \left(e^{-at_o} \left[\frac{\beta^2}{a^2 + \beta^2} e^{-a(t - t_o)} + \frac{a\beta}{a^2 + \beta^2} \sin \beta(t - t_o) \right.\right.$$

$$\left. - \frac{\beta^2}{a^2 + \beta^2} \cos \beta (t - t_o) \right] - e^{-bt_o} \left[\frac{\beta^2}{b^2 + \beta^2} e^{-b(t - t_o)} \right.$$

$$\left. + \frac{b\beta}{b^2 + \beta^2} \sin \beta(t - t_o) - \frac{\beta^2}{b^2 + \beta^2} \cos \beta(t - t_o) \right]$$

$$\left. - \frac{Q}{P} [1 - \cos \beta(t - t_o)] \right) \tag{66}$$

After a time t_r, the leaf has rotated to its arming position θ_r. The time T that it requires for the leaf to arm, once it has been released, is then

$$T = t_r - t_o \tag{67}$$

If these substitutions are made in equation (66), the arming position angle θ_r is obtained as a function of T and t_o.

$$\theta_r = \frac{P}{\beta^2}\Bigg(e^{-at_o}\left[\frac{\beta^2}{a^2+\beta^2} e^{-aT} + \frac{a\beta}{a^2+\beta^2}\sin\beta T - \frac{\beta^2}{a^2+\beta^2}\cos\beta T\right]$$

$$- e^{-bt_o}\left[\frac{\beta^2}{b^2+\beta^2} e^{-bT} + \frac{b\beta}{b^2+\beta^2}\sin\beta T - \frac{\beta^2}{b^2+\beta^2}\cos\beta T\right]$$

$$- \frac{Q}{P}\left[1 - \cos\beta T\right]\Bigg) \tag{68}$$

Since θ_r is also considered constant in this analysis, (68) is an equation relating the arming time T to the release time t_o, and can be expressed as an implicit function.

$$0 = e^{-at_o}\left[\frac{\beta^2}{a^2+\beta^2} e^{-aT} + \frac{a\beta}{a^2+\beta^2}\sin\beta T - \frac{\beta^2}{a^2+\beta^2}\cos\beta T\right]$$

$$-e^{-bt_o}\left[\frac{\beta^2}{b^2+\beta^2} e^{-bT} + \frac{b\beta}{b^2+\beta^2}\sin\beta T - \frac{\beta^2}{b^2+\beta^2}\cos\beta T\right]$$

$$- \frac{Q}{P}\left[1 - \cos\beta T\right] - \frac{\beta^2\theta_r}{P} \tag{69}$$

or

$$0 = f(t_o, T)$$

Though an analytic solution for T cannot be derived, graphical solutions can be achieved with the HDL analog computer. The computer solution is most conveniently obtained from the total derivative dT/dt_o, which is given by equation(D-9,)

$$\frac{dT}{dt_o} = \frac{e^{-at_o}[q_2 e^{-aT} + q_1 \sin\beta T - q_2\cos\beta T] - e^{-bt_o}[q_5 e^{-bT} + q_4 \sin\beta T - q_5 \cos\beta T]}{e^{-at_o}[-q_2 e^{-aT} + q_3 \sin\beta T + q_2 \cos\beta T] - e^{-bt_o}[-eq_5 e^{-bT} + q_6 \sin\beta T + q_5 \cos\beta T] - \frac{Q}{P}\sin\beta T} \tag{70}$$

where

$$q_1 = \frac{a^2}{a^2+\beta^2}\ ;\ q_2 = \frac{a\beta}{a^2+\beta^2};\ q_3 = \frac{\beta^2}{a^2+\beta^2}$$

$$q_4 = \frac{b^2}{b^2+\beta^2}\ ;\ q_5 = \frac{b\beta}{b^2+\beta^2}\ ;\ q_6 = \frac{\beta^2}{b^2+\beta^2} \tag{71}$$

To eliminate division by the computer (since the divisor approaches zero), the derivative dt/dt_o is replaced by (ref 7)

$$\frac{dT}{dt_o} = \frac{\frac{dT}{d\tau}}{\frac{dt_o}{d\tau}}$$

where τ is the machine time. Then (70) can be written (72)

$$\frac{dT}{d\tau} = \frac{e^{-at_o}[q_2 e^{-aT} + q_1 \sin\beta T - q_2\cos\beta T] - e^{-bt_o}[q_5 e^{-bT} + q_4 \sin\beta T - q_5 \cos\beta T]}{e^{-at_o}[-q_2 e^{-aT} + q_3 \sin\beta T + q_2 \cos\beta T] - e^{-bt_o}[-q_5 e^{-bT} + q_6 \sin\beta T + q_5 \cos\beta T] - \frac{Q}{P}\sin\beta T} \cdot \frac{dt_o}{d\tau} \tag{73}$$

Now division can be avoided by letting

$$\frac{dt_o}{d\tau} = e^{-at_o}[-q_2 e^{-aT} + q_3 \sin\beta T + q_2 \cos\beta T] - e^{-bt_o}[-q_5 e^{-bT} + q_6 \sin\beta T + q_5 \cos\beta T] - \frac{Q}{P}\sin\beta T$$

The computer programming is simplified by the generation of the subfunctions which appear in the expressions for $\frac{dT}{d\tau}$ and $\frac{dt_o}{d\tau}$. Each of these subfunctions is generated as the result of the machine solution of a simple first-order differential equation with rational coefficients. Thus, the following differential equations will generate the required

functions shown at the right;

$$\left.\begin{aligned}
\frac{df_1}{d\tau} &= -\,af_1\frac{dt_o}{d\tau} \quad ; \quad f_1 = e^{-at_o} \\
\frac{df_2}{d\tau} &= -bf_2\,\frac{dt_o}{d\tau} \quad ; \quad f_2 = e^{-bt_o} \\
\frac{df_3}{d\tau} &= -af_3\,\frac{dT}{d\tau} \quad ; \quad f_3 = e^{-aT} \\
\frac{df_4}{d\tau} &= -bf_4\,\frac{dT}{d\tau} \quad ; \quad f_4 = e^{-bT} \\
\frac{df_5}{d\tau} &= \beta f_6\,\frac{dT}{d\tau} \quad ; \quad f_5 = \sin \beta T \\
\frac{df_6}{d\tau} &= -\beta f_5\,\frac{dT}{d\tau} \quad ; \quad f_6 = \cos \beta T
\end{aligned}\right\} \qquad (74)$$

When these subfunctions are substituted into the expressions for $\frac{dT}{d\tau}$ and $\frac{dt_o}{d\tau}$ given by equations (72) and (73),

$$\left.\begin{aligned}
\frac{dT}{d\tau} &= f_1\,[q_2 f_3 + q_1 f_5 - q_3 f_6\,] - f_2\,[q_5 f_4 + q_4 f_5 - q_5 f_6\,] \\
\frac{dt_o}{d\tau} &= f_1\,[-q_2 f_3 + q_3 f_5 + q_2 f_6\,] - f_2\,[-q_5 f_4 + q_6 f_5 + q_5 f_6\,] - \frac{Q}{P}\,f_6\,,
\end{aligned}\right\} \qquad (75)$$

a complete set of equations (74) and (75) is obtained which, when programmed on the analog computer, can be solved graphically to show the variation in arming time T with release time t_o. The unscaled

flow diagram is shown in figure 14.

The initial conditions $(t_o, T)_{T=0}$ required for the integrating amplifiers of the computer must satisfy the implicit function (69) relating T to t_o. The earliest release time possible is t_{01}, which is obtained from a graphical solution of figure 15 plotted as

$$e^{-at_{o1}} - e^{-bt_{o1}} = \frac{C_2 N_{o1}}{C_1 A_o} = \frac{C_2 M_o}{C_1 m_i g y A_o} \tag{76}$$

This is shown in figure 15 for two acceleration curves. The most convenient method of determining the initial value of T is from a computer solution of equation (65), plotting θ versus time. T_1 is then the time for θ to rotate to θ_r, when it is released at time t_{01}. The form of (65) which is programmed is

$$\frac{d^2\theta_1}{d\tau^2} = -\beta_1^2\,\theta_1 + h(\tau - t_{o1})\,[P(f_1 - f_2) - Q_1] \tag{77}$$

where

$$\frac{df_1}{d\tau} = -af_1 \;;\quad f_1 = e^{-a\tau} \tag{78}$$

$$\frac{df_2}{d\tau} = -bf_2 \;;\quad f_2 = e^{-b\tau}$$

Its computer flow diagram is shown in figure 16. Relay amplifier No. 1 serves as a step function in this program by switching on the impressed function within the brackets at time t_{01}. The values obtained for T_1 from this program, together with t_{01}, enable initial conditions to be calculated for all the integrating amplifiers for the program of figure 14, including the initial values of the subfunctions. The following constants were used in the computer analysis:

$y = 0.0645$ in.

$k^2 = 0.0178$ in.^{2}

$C_1 = 1$

$C_2 = 1$

$M_o = 0.380$ ozf-in.

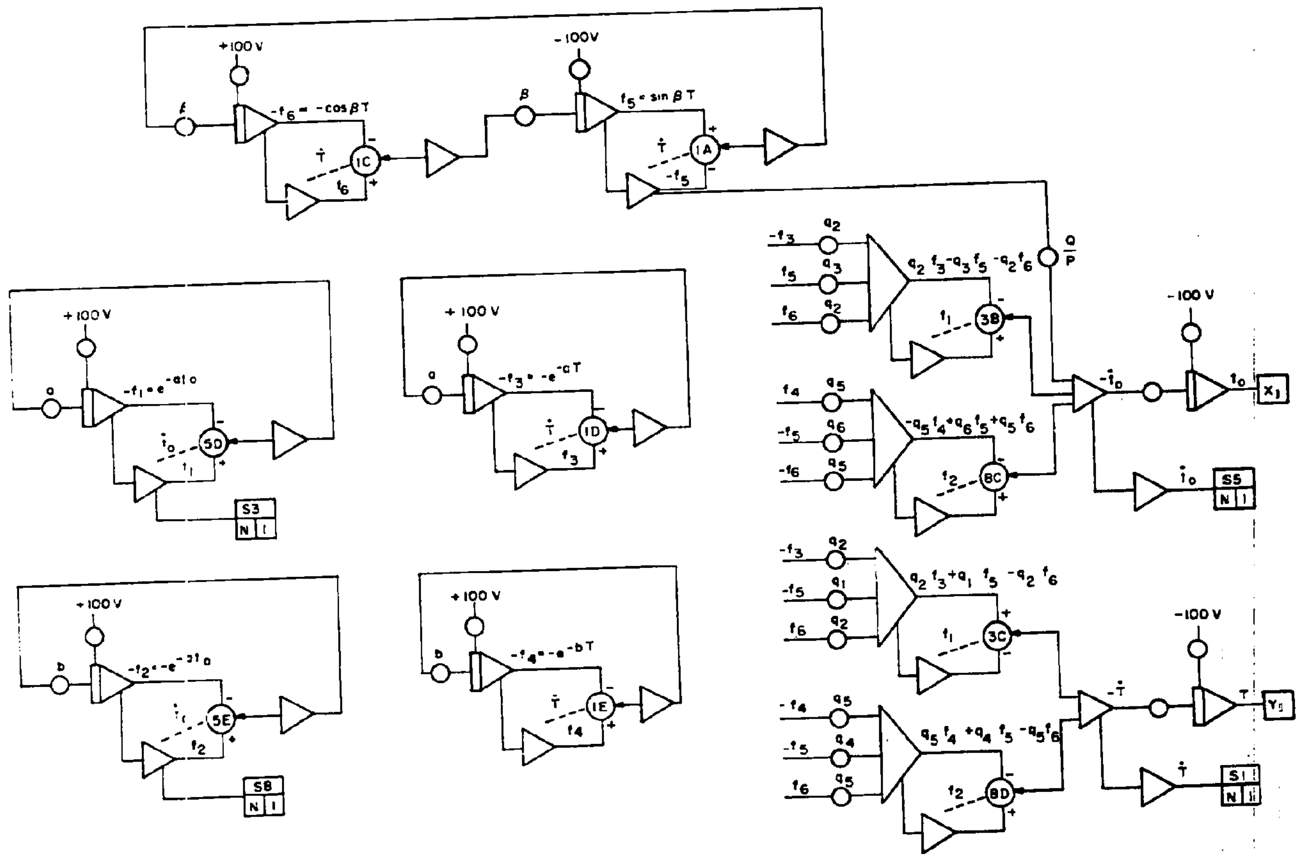

Figure 14. Unscaled computer circuit diagram for obtaining arming time T versus release time t_o.

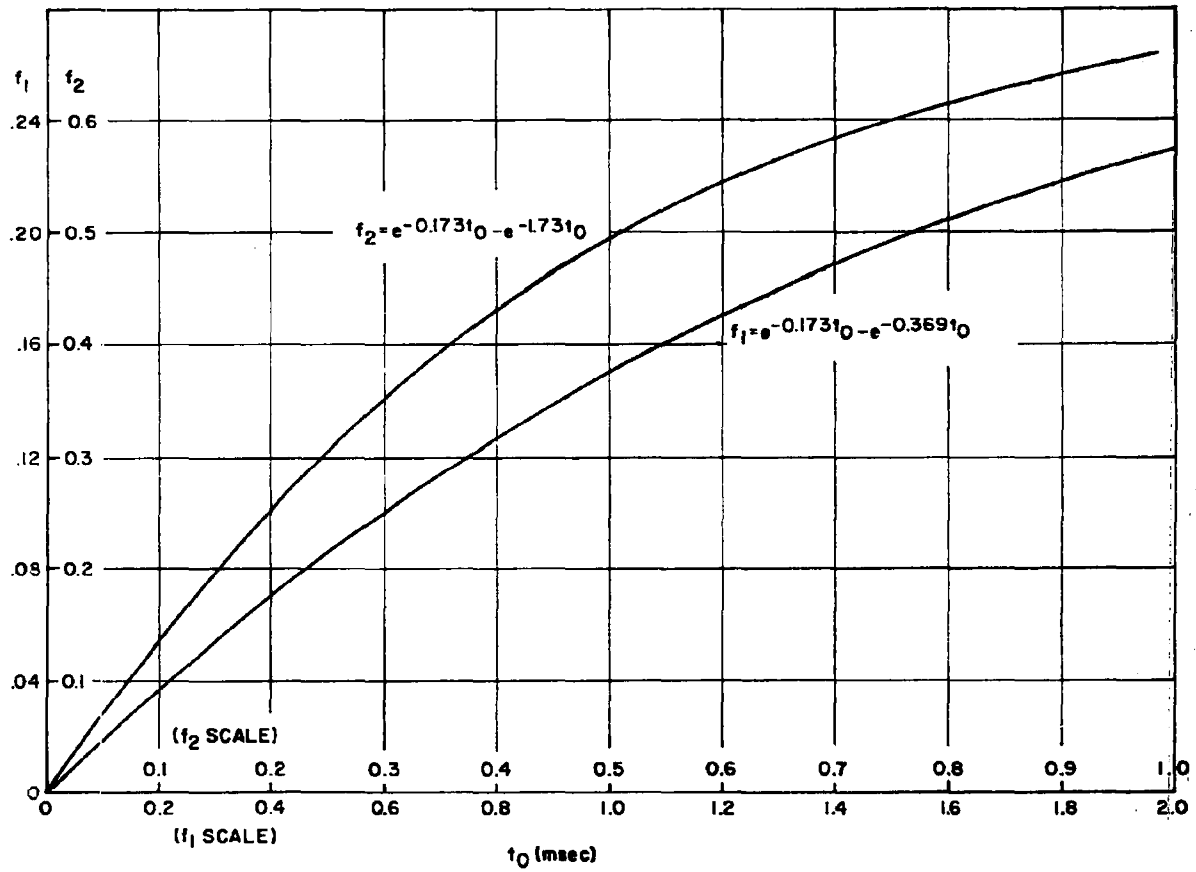

Figure 15. Curves for determining release time t_o of first leaf.

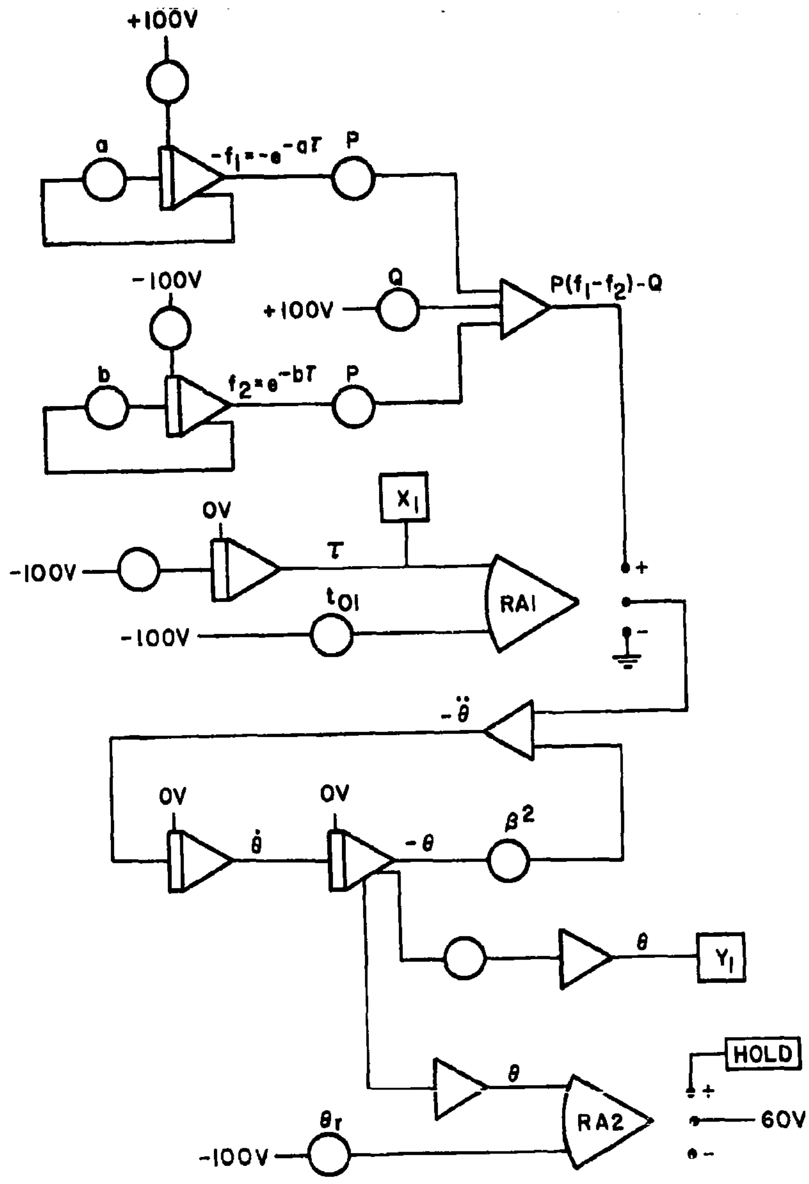

Figure 16. Unscaled circuit diagram for obtaining arming time of first leaf.

$$\theta_r = 45^\circ = 0.785 \text{ radians}$$

The fractional change is spring torque, $z = \frac{\lambda\theta_r}{M_o}$, was assigned the two-values 0.4 and 2.0, the first representing a typical setback-leaf spring stiffness and the second, a very steep spring rate. The major variable in the computer runs was the mass, which was expressed in equation (56)

$$m = m_r m_o \tag{79}$$

where m_o is 0.003686 $\frac{ozf}{g}$, and m_r is a dimensionless decimal multiplier. Values of m_r that were used varied from 2.5 to 12.0.

Two different gun acceleration functions were studied. The first, which has a medium fast rise, was a good approximation to the T28E6 mortar (as discussed in appendix E):

$$A(t) = 2910(e^{-0.173t} - e^{-0.369t}) \tag{80}$$

The peak of this curve is 792 g, which occurs after 3.86 msec. The second acceleration curve studied has the same peak value with a more steeply rising front, reaching the peak in only 1.48 msec.

$$A(t) = 1136(e^{-0.173t} - e^{-1.73t}) \tag{81}$$

In both of these expressions, time is in milliseconds. Thus, for the first acceleration curve, $A_o = 2910$, a = 0.173 per msec, b = 0.369 msec, and $\eta = \frac{b}{a} = 2.133$. For the second curve $A_o = 1136$, a = 0.173 per msec, b = 1.73 per msec, and $\eta = 10$.

Three sets of computer solutions for the arming time as a function of the release time were obtained. The first set, shown in figure 17, was developed for the case of z = 0.4 and the acceleration function given by equation (80). For this case, the constants in the computer programs had the following values: P = 4.060 per msec^2 $Q = \frac{2.232}{m_r}$ per msec^2, $\beta^2 = 2.84 \frac{z}{m_r} - \frac{1.136}{m_r}$ per msec^2, and t_{01} was the time at which the curve of figure 15 reached the value

$$\frac{C_1 M_o}{C_2 m_r m_o g y A_o} = \frac{0.5484}{m_r}.$$

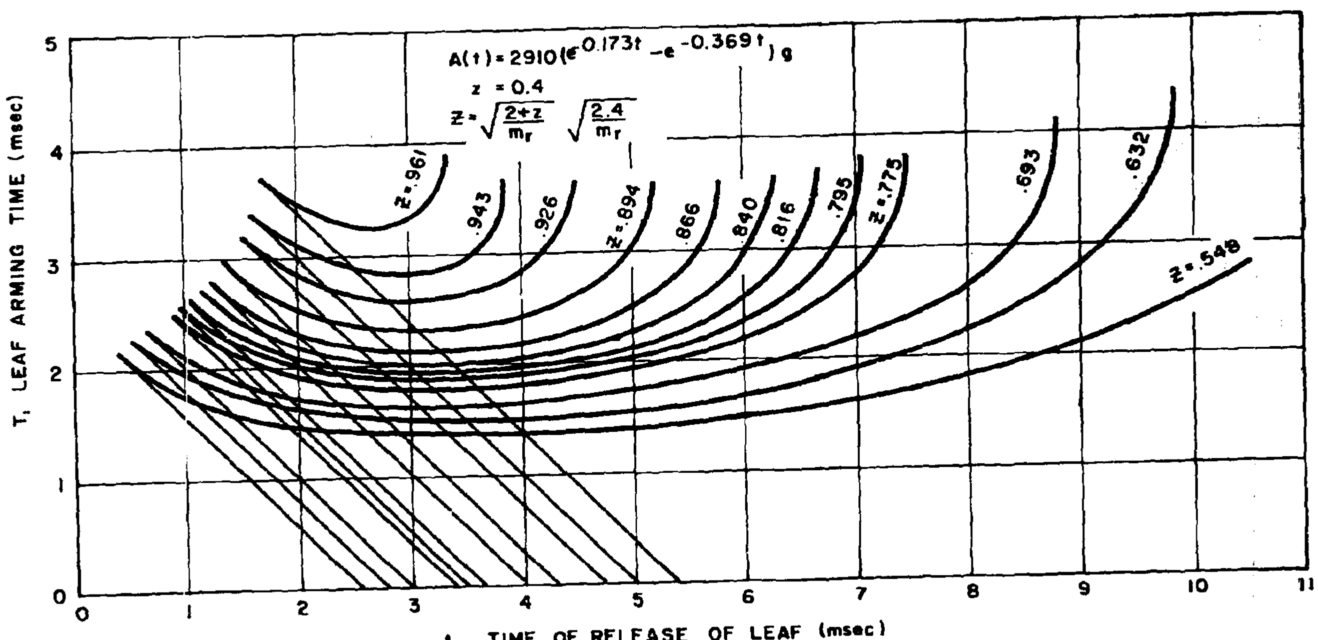

Figure 17. Arming time of leaves of different mass—Case I.

The second set differs from the first in that the fast rise acceleration given by equation (81) is now used instead of (80). Now Q and β^2 are the same as before, but $P = 1{,}587$ per msec^2 and t_{01} is now the time at which the curve reaches $\frac{1.4048}{m_r}$. These runs are shown in figure 18.

The third and final set, shown in figure 19, differed from the first in that now the relative change in spring torque was increased from 0.4 to 2.0. This change affected only β^2, the other constants remaining the same; β^2 now equaled $\frac{5.68}{m_r}$ per msec^2.

The curves of figures 17, 18, and 19 demonstrate the variation in the arming time as a function of the release time while the mass of the leaf changes. Although the mass of the leaf is the parameter of the curves, the numbers associated with each trace in these figures represent the drop-safety index $Z = \left(\frac{2 + z}{m_r}\right)^{\frac{1}{2}}$. Since z is a constant for each set of curves, the index depends only on the mass. This index is the parameter of importance for each leaf since the sum of the individual indices measures the relative safety of the multiple leaf device. From these sets of curves it is possible to select the combination of leaf masses that will at the same time both arm under the given acceleration function and provide the greatest safety, as measured by the drop-safety index.

6. OPTIMIZATION OF THE DROP-SAFETY INDEX

The selection of a set of leaf masses that will arm from the curves of figures 17, 18, and 19 is both simple and straightforward. Each trace represents a separate mass, and there are enough traces to determine the arming time of any given mass. The actual procedure, however, is really to find the arming time for a leaf of a particular index of drop safety, and then determine the corresponding mass from the index by equation (58)

$$m_1 = \frac{(2 + z)\ m_0}{Z^2} \tag{82}$$

Each trace has a particular mass and drop-safety index associated with it, and each set of traces is for one particular z and one acceleration function only. Thus, figure 17 was obtained for $z = 0.4$ and acceleration function (80), figure 18 for $z = 0.4$ and function (81), and figure 19 for $z = 2.0$ and function (80).

It will be noted that the larger the safety index (and the smaller the mass) of a leaf, the longer is the arming time. Each

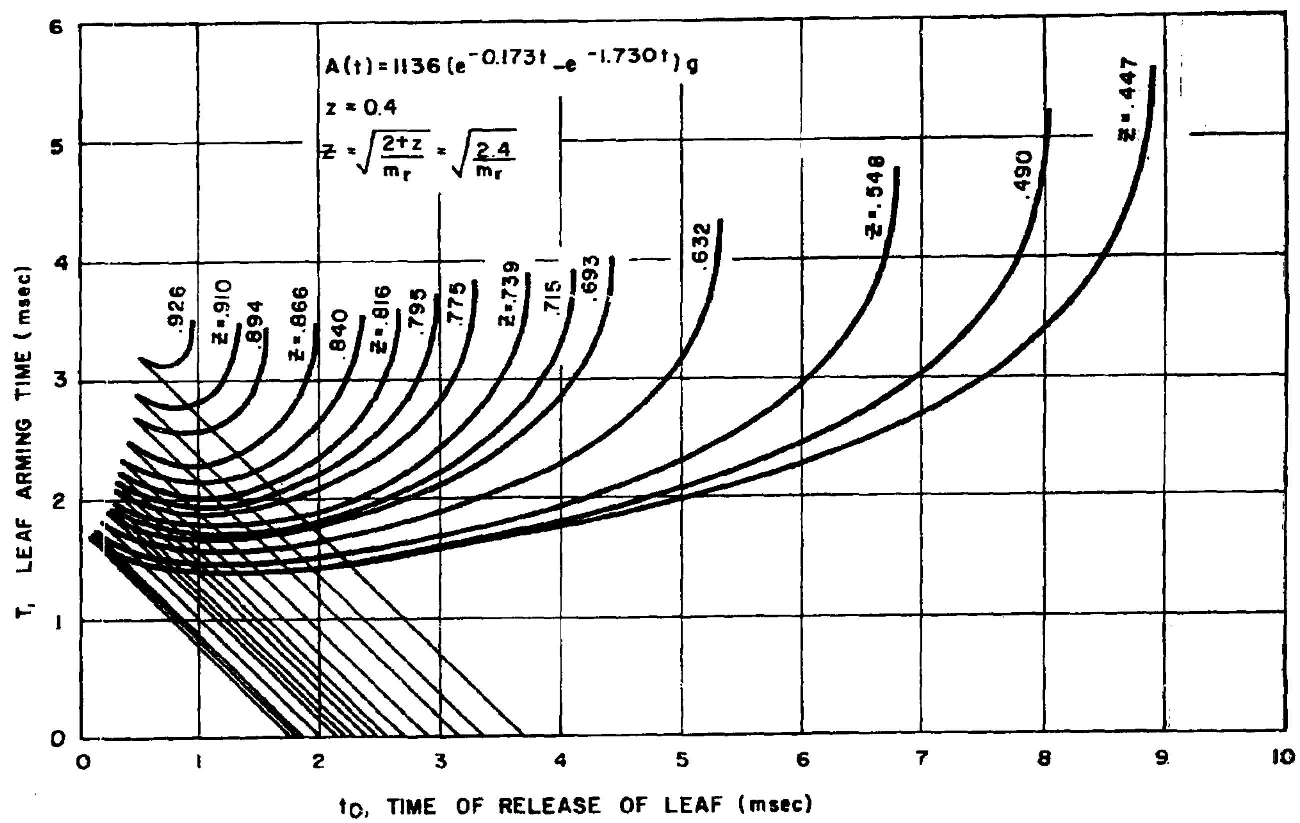

Figure 18. Arming time of leaves of different mass—Case II.

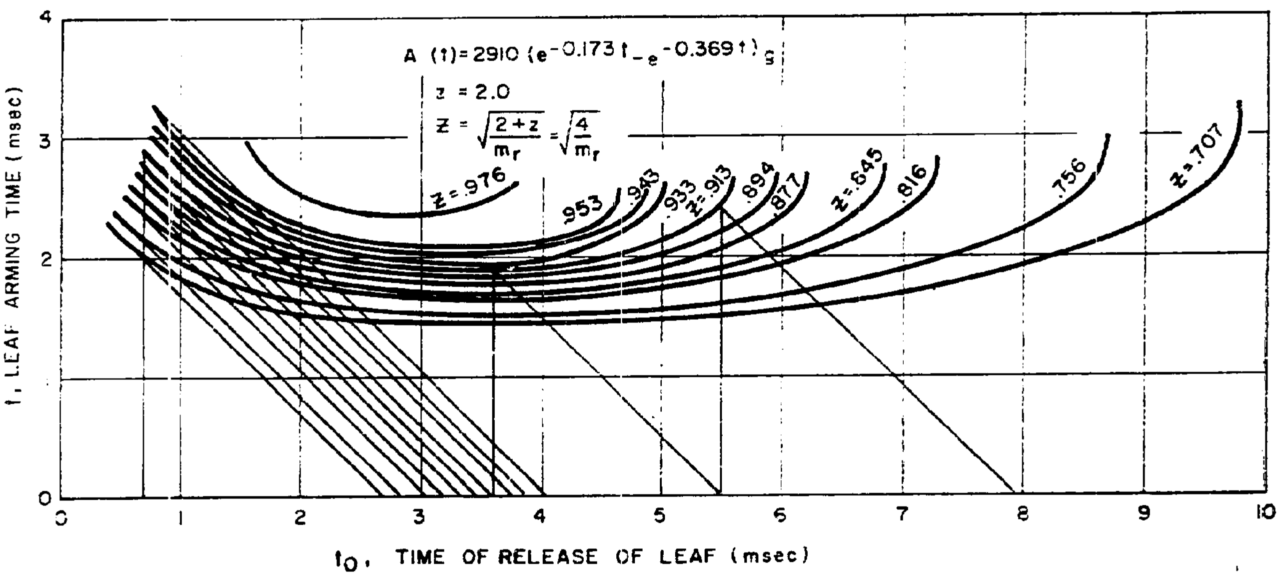

Figure 19. Arming time of leaves of different mass—Case III.

trace has definite starting and ending points; these define the limits of the release time between which the particular leaf will arm. In addition, the leaves with the larger indices take longer to start rotating because of larger N_{oi}'s. It is seen from these curves that there is a limit to the amount of time that a leaf takes to arm. For instance, figure 17 indicates that all but the heaviest leaves arm in under 4 msec. In the case of figures 18 and 19 this upper limit seems to be about 3.3 and 4 msec, respectively. This means that a leaf, if it is going to arm at all, will do so within a certain limited time regardless of when it is released. This results from the fact that the leaf-spring combinations have natural periods of oscillation.

The combined total of the starting time of the first leaf and its arming time, $(t_{o1}+T_1 \doteq t_{r1})$, is indicated by the terminus of the 45-deg dashed lines drawn down to the right from the starting point of each trace. The terminal points of each of these lines are the times at which the first leaf is armed, and correspondingly, the second leaf released.

Now if this new release time for the second leaf is used as the horizontal coordinate, the arming time for leaves of different indices can be obtained from the curves. This time is added to the release time to obtain the accumulated time until the second leaf is armed.

Since this is now the release time of the third and last leaf, it is desirable to choose as the third leaf the one with the largest drop-safety index that will still arm. This will be the topmost trace that extends as far as the release time of the third leaf. The arming time for this leaf added to its release time is the total arming time for the three-leaf device. (Since the third leaf just reaches arming position, it might be thought that this is a poor choice, since no allowance was made for manufacturing tolerances, etc. However, each combination is being obtained for comparison with other combinations. The sum of the indices measures the relative safety of the three leaves selected.

This procedure can be repeated for various combinations of the particular indices, and the results compared. A useful systematic procedure is shown in table II for one combination from figure 17. Actually, this particular leaf combination illustrated had the highest total drop-safety index that could be found for any of the combinations of figure 17.

TABLE II. DETERMINATION OF ARMING TIME FOR A GIVEN LEAF COMBINATION

i	z_i	t_{oi}	T_i	$t_{ri} = t_{oi} + T_i$	$m_{ri} = \frac{2+z}{z_i^2} = \frac{2.4}{z_i^2}$
1	0.926	1.49	3.22	4.71	2.8
2	0.894	4.71	2.72	7.43	3.0
3	0.775	7.43	3.40	10.83	4.0
	2.595				

$$Z = \sum_{i=1}^{3} z_i = 2.595$$

In determining the combinations with the largest total index, one can begin by selecting the largest index; i. e., 0.961, for the first leaf and then trying various combinations of indices for the last two leaves. The last leaf is always chosen to have as large an index as possible. Then, after these combinations have been totaled, the next step is to choose the leaf with the second largest index (trace) as the starter, and to try various combinations of second and third leaves with it. The process continues for the third largest index, the fourth, etc., as long as it is possible to obtain appreciable values of the total index. Actually, it soon becomes apparent what combinations have the larger values, making it unnecessary to try very many combinations, even through a large variety was calculated from figures 17, 18, and 19 for demonstration purposes. These values obtained for the total index for various combinations of m_r are shown in tables III, IV, and V, respectively. Also given is the time required for the leaves to finish arming. In table VI are given equivalent values of m_r and Z for z = 0.4 and 2.0.

The results given in tables III, IV, and V are presented in two groups. At the left are listed the indices obtained for combinations of leaf masses where the mass of the first leaf is less than or equal to the mass of the second; at the right are listed the combinations where the second leaf is heavier than the first. On the same line are the cases where values of m_1 and m_2 are interchanged.

An examination of these results, for the case of figure 17 and table III indicates the optimum value of the index is 2.595. This index value is obtained with relative leaf masses of 2.8, 3.0, 4.0, or 3.0, 2.8, 4.0, the second combination being the same as the first except for the interchange of the first and second leaves. There are numbers of other combinations in table III whose index is within 1 percent

TABLE III. DETERMINATION OF DROP-SAFETY INDEX—CASE I

$m_1 \le m_2$					$m_1 > m_2$				
m_1	m_2	m_3	t_{r3}	Z	m_1	m_2	m_3	t_{r3}	Z
2.6	3.2	5.0	10.90	2.520					
2.6	4.0	4.0	10.80	2.511					
					5.0	2.6	3.6	9.28	2.470
2.7	3.0	5.0	10.66	2.530					
2.7	3.2	5.0	9.80	2.502					
2.7	3.6	4.0	10.03	2.534	3.6	2.7	3.6	10.16	2.575
2.7	3.8	3.8	10.53	2.533	3.8	2.7	3.6	9.36	2.554
2.7	4.0	3.8	10.06	2.513	4.0	2.7	3.6	9.01	2.534
2.7	5.0	3.8	9.58	2.431	5.0	2.7	3.6	8.29	2.452
2.8	3.0	4.0	10.83	2.595	3.0	2.8	4.0	10.80	2.595
2.8	3.2	3.8	10.77	2.587	3.2	2.8	3.8	9.92	2.587
2.8	3.6	3.8	9.69	2.537	3.6	2.8	3.6	9.14	2.558
2.8	3.8	3.6	10.07	2.537	3.8	2.8	3.6	8.79	2.537
					4.0	2.8	3.6	8.55	2.517
					5.0	2.8	3.2	8.59	2.485
3.0	3.0	3.8	9.82	2.583					
3.0	3.2	3.6	9.70	2.576	3.2	3.0	3.6	9.42	2.576
3.0	3.6	3.6	9.10	2.526	3.6	3.0	3.6	8.54	2.526
					3.8	3.0	3.6	8.29	2.505
					4.0	3.0	3.2	8.96	2.535
3.2	3.2	3.6	8.92	2.548					
3.2	3.6	3.6	8.52	2.498	3.6	3.2	3.2	9.29	2.548
					3.8	3.2	3.2	8.70	2.527
					4.0	3.2	3.2	8.37	2.507
3.6	3.6	3.2	8.55	2.498					
3.4	3.4	3.4	8.52	2.520					
2.6	3.9	5.2	9.71	2.426					
					3.8	3.6	3.2	8.20	2.477
					6.0	2.6	3.6	8.70	2.409
					6.0	2.7	3.2	8.68	2.441
					8.0	2.6	3.24	9.32	2.369
					8.0	2.7	3.2	8.16	2.357

TABLE IV. DETERMINATION OF DROP-SAFETY INDEX—CASE II

$m_1 \leq m_2$					$m_1 > m_2$				
m_1	m_2	m_3	t_{r3}	Z	m_1	m_2	m_3	t_{r3}	Z
2.8	4.4	9.0	11.7	2.185					
2.8	5.0	7.5	10.23	2.189					
2.8	6.0	6.5	9.85	2.167					
2.9	4.4	7.0	10.31	2.234					
2.9	5.0	6.5	9.67	2.211					
2.9	6.0	6.3	9.37	2.167					
3.0	4.0	7.5	10.27	2.239					
3.0	4.4	6.6	9.72	2.243					
3.0	5.0	6.1	9.55	2.217					
3.0	6.0	5.7	9.10	2.176					
3.2	3.8	7.0	10.28	2.246					
3.2	4.0	6.5	9.78	2.256					
3.2	4.4	6.0	8.89	2.237					
3.2	5.0	5.7	8.56	2.209					
3.2	6.0	5.5	8.39	2.158					
					8.0	3.2	5.6	8.63	2.074
3.4	3.8	6.3	9.51	2.255					
3.4	4.0	6.0	8.82	2.247					
3.4	4.4	5.7	8.54	2.228	4.4	3.4	6.2	9.45	2.204
3.4	5.0	5.4	8.50	2.203	5.0	3.4	5.5	8.95	2.193
					6.0	3.4	5.3	8.43	2.147
3.6	3.6	6.4	9.72	2.242					
3.6	3.8	5.8	8.82	2.251	3.8	3.6	5.9	9.39	2.236
3.6	4.0	5.6	8.59	2.246	4.0	3.6	6.0	8.50	2.223
3.6	4.4	5.3	8.33	2.230	4.4	3.6	5.5	8.57	2.215
					5.0	3.6	5.3	8.40	2.184
					6.0	3.6	5.0	7.86	2.141
3.8	3.8	5.7	9.00	2.240					
3.8	4.0	5.5	8.62	2.230	4.0	3.8	5.5	8.64	2.230
3.8	4.4	5.2	8.39	2.214	4.4	3.8	5.2	8.43	2.209
3.8	5.0	5.0	7.99	2.181	5.0	3.8	5.0	8.11	2.181
					6.0	3.8	4.8	7.79	2.132
4.0	4.0	5.3	8.46	2.225					
4.0	4.4	5.0	8.45	2.207	4.4	4.0	5.0	8.48	2.207
4.0	5.0	4.8	7.86	2.173	5.0	4.0	4.8	7.88	2.173
4.4	4.4	4.9	7.88	2.178					
4.4	5.0	4.7	7.71	2.142	5.0	4.4	4.7	7.70	2.142
4.7	4.7	4.7	7.69	2.145					
5.0	5.0	4.6	7.55	2.111					

TABLE V. DETERMINATION OF DROP-SAFETY INDEX—CASE III

$m_1 \le m_2$					$m_1 > m_2$				
m_1	m_2	m_3	t_{r3}	Z	m_1	m_2	m_3	t_{r3}	Z
					4.6	4.2	5.4	8.88	2.770
					4.8	4.2	5.1	8.66	2.775
4.4	4.4	5.2	8.87	2.783					
					4.5	4.4	5.1	8.52	2.782
					4.6	4.4	5.0	8.44	2.780
					4.8	4.4	4.9	8.19	2.770
4.4	5.0	5.0	8.53	2.741					
4.4	6.6	8.8	7.06	2.406					
4.5	4.6	5.0	8.46	2.770					
					4.8	4.5	4.9	8.09	2.760
4.6	4.6	4.9	8.38	2.770					
4.8	4.8	4.8	7.95	2.739					

TABLE VI. DROP-SAFETY INDEX FOR EACH MASS

m_r	$Z = \left(\frac{2.4}{m_r}\right)^{\frac{1}{2}}$
2.4	1.000
2.6	.961
2.7	.943
2.8	.926
2.9	.910
3.0	.894
3.2	.866
3.4	.840
3.6	.816
3.8	.795
4.0	.775
4.4	.739
4.7	.715
5.0	.693
6.0	.632
8.0	.548
10.0	.490
12.0	.400

m_r	$Z = \left(\frac{4.0}{m_r}\right)^{\frac{1}{2}}$
4.0	1.000
4.2	.976
4.4	.953
4.5	.943
4.6	.933
4.7	.923
4.8	.913
5.0	.894
5.2	.877
5.4	.861
5.6	.845
5.8	.830
6.0	.816
7.0	.756
8.0	.707

of the optimum value such as:

m_{r1}	m_{r2}	m_{r3}
2.8	3.2	3.8
3.0	3.0	3.8
3.0	3.2	3.6
3.6	2.7	3.6
3.2	2.8	3.8
3.2	3.0	3.6

The masses are all about the same size, differing very little in magnitude. In general, for this case, the index will be slightly larger if the first mass is heavier than the second. This probably results from the fact that the average applied gun acceleration is less during the rotation of the first leaf than the second.

If the leaves are all of equal mass ($m_1 = m_2 = m_3 = 3.4$), a drop-safety index of 2.520 can be obtained, which is only 3 percent less than the optimum value possible by varying the masses.

If a 2, 3, 4 ratio of the leaf masses is used--as has been a common rule-of-thumb choice in the design of setback leaf devices--the largest index that can be obtained is 2.426 for a 2.6, 3.9, 5.2 combination. This index is 6-1/2 percent less than the optimum and 4 percent less even than that obtained with equal masses.

For the case of figure 17 and table IV (the steeply rising acceleration), the optimum index appears to be 2.256 from a 3.2, 4.0, 6.5 combination. Other combinations whose index is within 1 percent of the optimum are:

m_{r1}	m_{r2}	m_{r3}	m_{r1}	m_{r2}	m_{r3}
2.9	4.4	7.0	3.6	3.6	6.4
3.0	4.0	7.5	3.6	3.8	5.8
3.0	4.4	6.6	3.6	4.0	5.6
3.2	3.8	7.0	3.8	3.8	5.7
3.2	4.4	6.0	3.8	3.6	5.9
3.4	3.8	6.3			
3.4	4.0	6.0			

In this case, the 2, 3, 4 ratio of leaf masses is obviously more nearly optimum; for instance, a 3.2, 4.8, 6.4 ratio would have an index of about 2.200 (interpolating between curves), which is only 2 1/2 percent less than optimum. A ratio of 2, 2, 3 would be an even better choice. The index will, in general, be larger now when the first leaf is lighter than the second, the average applied acceleration being greatest during the first leaf rotation.

For leaves of equal mass (4.7, 4.7, 4.7), an index of 2.145 is obtained, which is only 5 percent less than optimum.

For the third and final case of the steep spring rate (figure 19 and table V), the optimum index was found to be 2.783 from a 4.4, 4.4, 5.2 combination of relative leaf masses. Again, there are a number of other combinations within 1 percent of this index:

m_{r_1}	m_{r_2}	m_{r_3}	m_{r_1}	m_{r_2}	m_{r_3}
4.5	4.6	5.0	4.5	4.4	5.1
4.6	4.6	4.9	4.6	4.4	5.0
4.6	4.2	5.4	4.8	4.4	4.9
4.8	4.2	5.1	4.8	4.5	4.9

Now, the higher values of the index occur for combinations of nearly equal masses. In fact, the 4.8, 4.8, 4.8 combination has an index only 1 1/2 percent less than the optimum. A ratio of 2, 3, 4, such as 4.4, 6.6, 8.8 masses, would obviously be a poor choice in this case.

It will be noted that the lightest leaf (the top trace of figure 19) would not arm if released when the acceleration equaled the equivalent initial spring torque N_1, but only if released after the acceleration had a chance to increase further. Thus, this lightest mass of (4.2) could not be selected for the first leaf, but only for the second or third leaf. It would not arm at first release because the average applied acceleration then was too small, the period of the natural frequency of oscillation of the mass-spring system being short.

For each of these cases there is obviously an infinite number of possible combinations of leaf masses that could be used. However, those particular masses that were tested represent an adequate sampling, since the discrete values were chosen close enough together so that no significant improvement (greater than a small fraction of 1 percent) could be obtained in the index by using intermediate values of relative mass.

7. RESULTS

The analyses of section 6 indicate that, in all cases, a set of equal masses for all three leaves in a setback mechanism can have a drop-safety index almost as large as the best set of unequal masses. The maximum drop-safety index that can be obtained by varying the mass of the leaves is only several percent higher than the index that can be found for three leaves of some equal mass. Therefore, for the sake of simplifying the design, it is recommended that all leaves be of the same mass since little is to be gained otherwise.

This analysis is based on a mathematical model in which it is assumed that the frictional load on the last leaf is equal to that on the other leaves. However, if the frictional load on the last leaf is unavoidably large--being a heavy latch or other device--it may not be possible to use leaves of equal mass, or at least leaves of the preferred mass. In this case, it might be desirable to design the mechanism using the detailed method of sections 5 and 6. In any case, it is advisable to keep the load on the last leaf to a minimum by the use of materials with low coefficients of friction and by optimum design of the element bearing on the last leaf.

When the values of the drop-safety index for the three cases are compared, it is found that the indices are lower for the case of the fast rising acceleration curve (Case II). This results because the available velocity change, or area under the acceleration curve, is less. The indices are a few percent larger for the case of the steep spring rate (Case II). However, the flatter spring rate is to be preferred because of tolerance problems in spring manufacturing, as discussed in section 3.

The optimum mass that the leaves should have is the smallest value that will still result in the mechanism arming during the applied gun acceleration. It will be demonstrated that this value can be obtained by averaging the applied gun acceleration that is available to arm the leaves in excess of some N_o, and comparing it with the constant acceleration required to arm in the same available time, assuming the same N_o. This process is repeated for several values of N_o until the average acceleration available is found equal to the constant acceleration required. This then determines the optimum value of N_o, and hence, of m. This procedure is essentially the same as the design procedure described in section 3, except that for this analysis the mass is the principal variable, the spring rate and initial spring torque being held constant.

The average acceleration available to arm is obtained by finding the area under the gun curve and above an arbitrarily selected acceleration N_o, and dividing by the time period that the gun acceleration exceeds N_o. Since a mathematical function was used for the gun acceleration, the average for each N_o can be obtained by integration.

$$\overline{A - N_o} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left[A_o \left(e^{-at} - e^{-bt}\right) - N_o \right] dt \tag{83}$$

$$= \frac{A_o}{t_2 - t_1} \left[\frac{e^{-at_1} - e^{-at_2}}{a} - \frac{e^{-bt_1} - e^{-bt_2}}{b} \right] - N_o$$

where t_1 and t_2 are the two times when the function equals N_o:

$$A_o(e^{-at_j} - e^{-bt_j}) = N_o;\ t_j = t_1,\ t_2 \tag{84}$$

The constant acceleration required to arm three identical leaves in the time $(t_2 - t_1)$ for each N_o is given by equation (18). If $z = \frac{\theta_r}{\theta_o}$ is substituted and C_1 and C_2 are set equal to unity, the constant acceleration to arm each leaf is

$$A - N_o = \frac{zN_o}{1 - \cos\sqrt{\frac{ygzN_o}{k^2\theta_r}}\ t_r} \tag{85}$$

where the time to arm each leaf is

$$t_r = \frac{t_2 - t_1}{3} \tag{86}$$

and

$$\beta = \sqrt{\frac{ygzN_o}{k^2\ \theta_r}} \tag{87}$$

The angle βt_r is limited to a maximum of π radians since the leaf rotation reaches its maximum at this value of βt_r. Therefore, if

$$\beta t_r = \sqrt{\frac{ygzN_o}{k^2\ \theta_r}}\ \frac{(t_2 - t_1)}{3}$$

is greater than π radians, t_2 must be reduced to such a time that βt_r just equals π. Then, since less time is available in which to arm the three leaves, $A - N_o$ and $A - N_o$ must be redetermined for this smaller t_2 substituted in (83) and (85). This special limiting of t_2 is only required for large values of z, as in Case II. However, the usual values of z are small. After the value of N_o is determined for which the available acceleration is equal to the constant acceleration required, the optimum mass of all the leaves is obtained from (13)

$$m = \frac{M_o}{ygN_o}$$

To test this design procedure, it was applied to the three cases of section 6, for which the optimum relative masses were, in order: 3.4, 4.7, and 4.8. The application of this simplified design procedure resulted in optimum relative masses of 3.3, 4.7, and 4.8. Thus, this technique is seen to be accurate for determining the optimum mass to provide maximum safety in a setback-leaf mechanism while still being designed to arm when subjected to the gun acceleration. Why this empirical method should work so well is not fully understood; however, it is probably because of an "averaging-out" of the mean arming time of each leaf over the rising and falling applied acceleration. This fact is partly illustrated by the results of appendix B, where it is seen that the arming time for increasing linear acceleration is longer than for a linear decreasing acceleration of the same total velocity change--the time for a constant acceleration being intermediate.

It is interesting to examine the instantaneous distribution of the applied acceleration torque between the spring torque and the inertial torque for each leaf of the mechanism. Figures 20, 21, and 22 exhibit these torques for the three cases of section 6 with equal leaf masses. These curves are analog computer solutions for each of the bracketed parts of the following equation:

$$\left[I \frac{d^2\theta_i}{dt^2}\right] + \left\lfloor \lambda(\theta_o + \theta_i) \right\rceil = h(t - t_{oi}) \left\lfloor myg\ A(t) \right\rceil \qquad (88)$$

or

$$M_m + M_s = M_A \qquad (89)$$

It will be noted in the first two cases that the spring torque on the third leaf exceeds the applied torque before the arming position is reached, so that the leaf's angular acceleration becomes negative. However, in the third case with the steep spring rate, the spring torques of all three leaves acquire enough momentum to surpass the applied torque before reaching arming positions. The spring torque curves vary as θ varies, and the inertial torque varies as the angular acceleration.

8. CONCLUSIONS

It is seen from the detailed methematical analysis that, wherever possible, it is desirable to design setback mechanisms with leaves of equal weight. A procedure for designing such mechanisms can be based on treating the applied acceleration as constant. Because of a lack

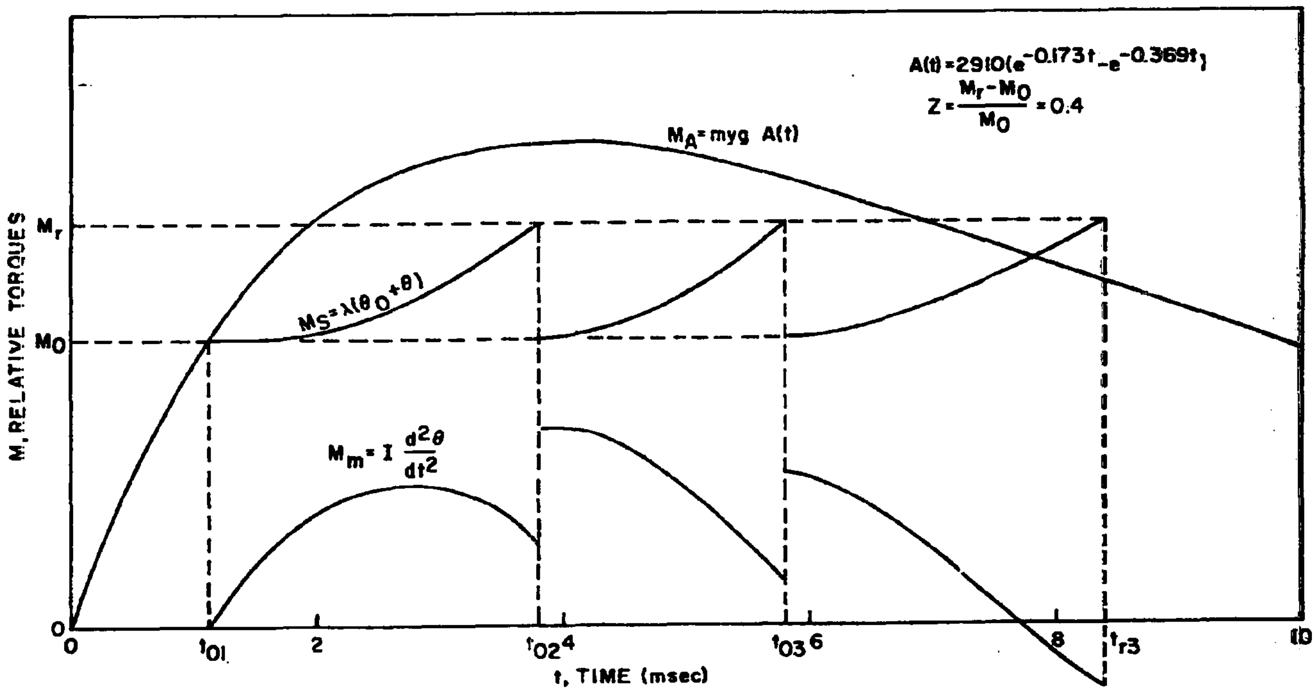

Figure 20. Relative distribution of applied acceleration torque between spring torque and inertial torque for case I.

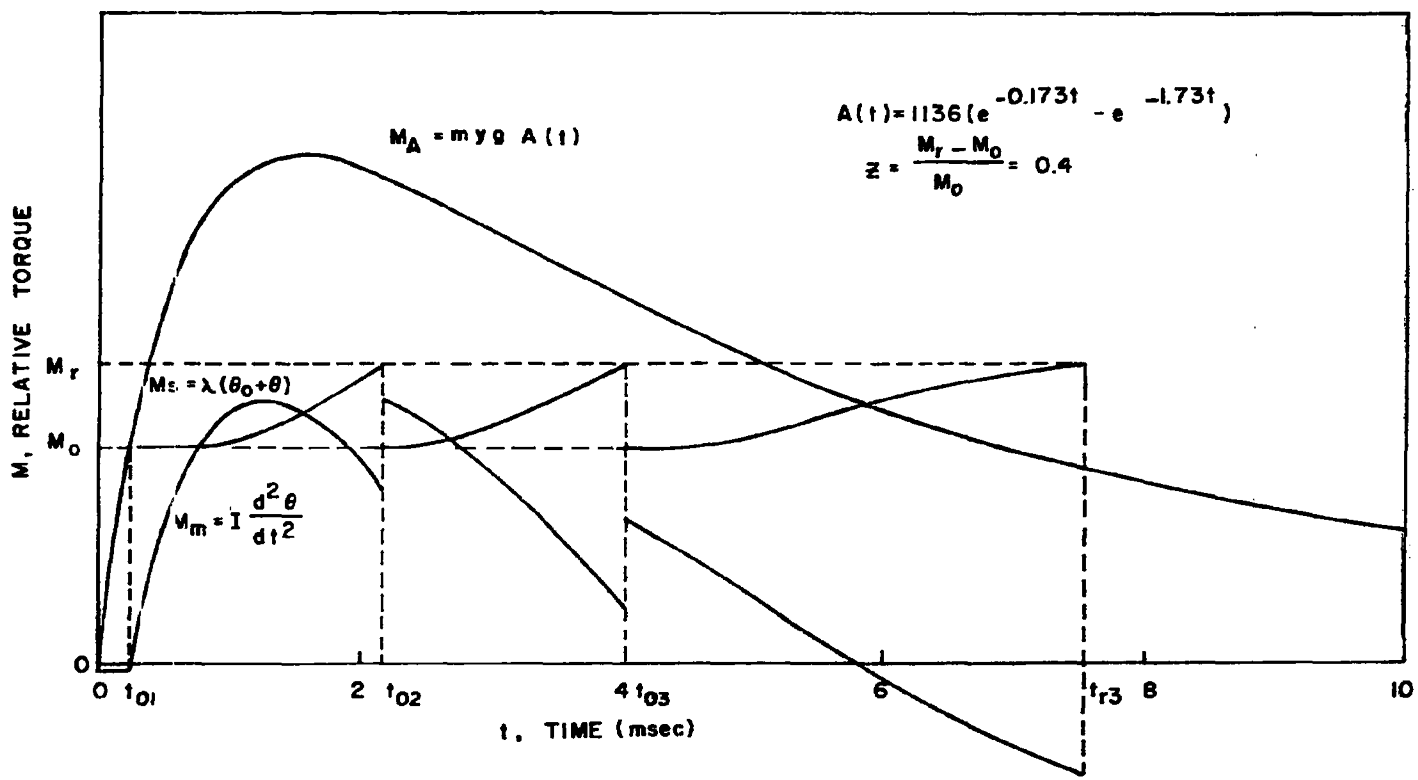

Figure 21. Relative distribution applied acceleration torque between spring torque and inertia torque for case II.

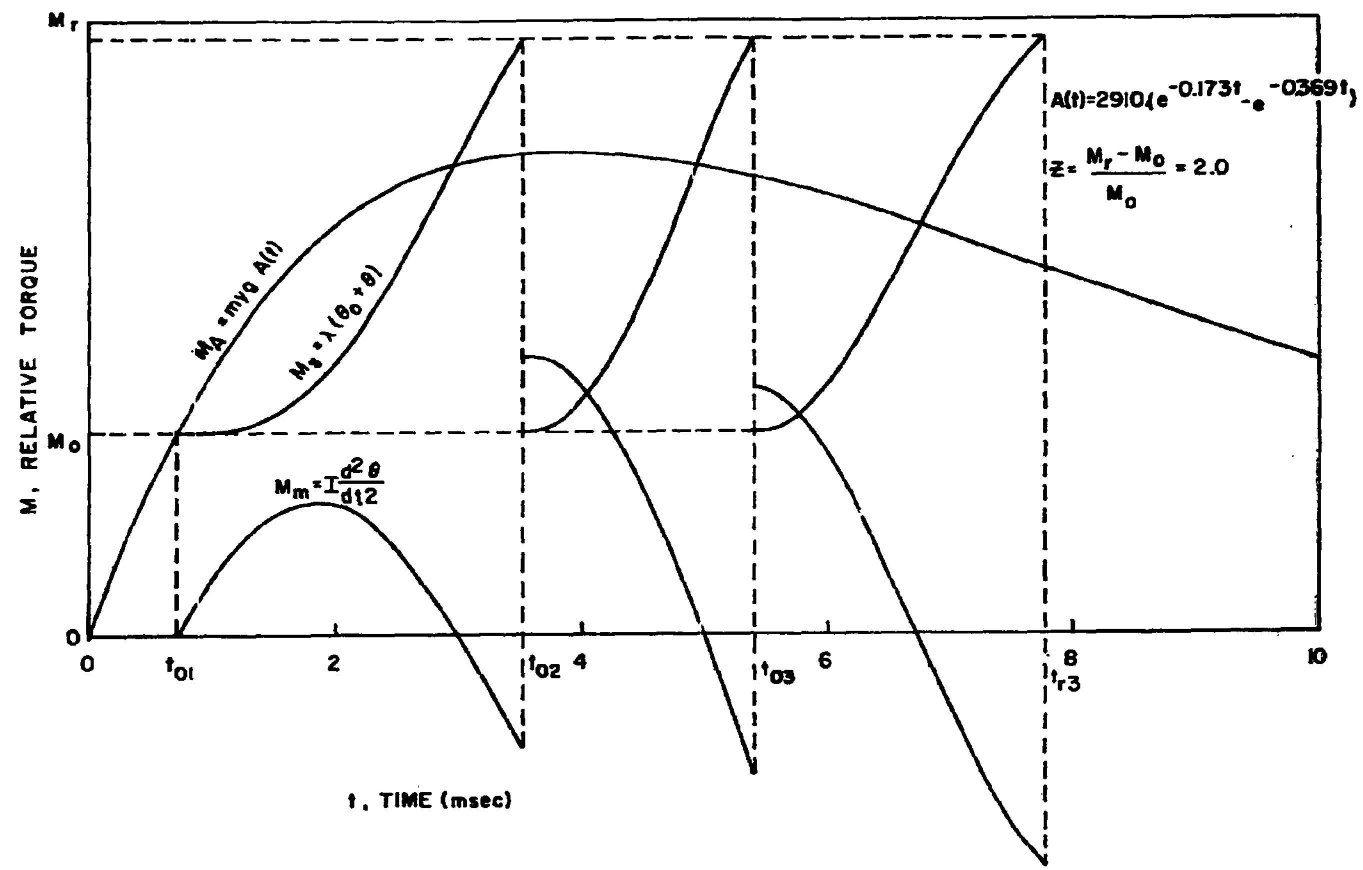

Figure 22. Relative distribution of applied acceleration torque between spring torque and inertial torque.

of information, friction was treated as a constant, although it is certain to vary for different environmental conditions and from unit to unit. Experimental results are needed to further evaluate this design procedure. Centrifuge-testing permits the application of constant accelerations, and the results could be used to determine the effective friction coefficient that should be allowed. Obviously, manufacturing techniques that reduce friction to a minimum should be used.

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*References 2, 3,and 4 are being prepared for publication as formal technical reports.

APPENDIX A

THE CENTER OF MASS AND MOMENT OF INERTIA OF A LEAF

The center of mass and moment of inertia of a representative leaf are calculated by parts. Since the leaf (figure A-1) is symmetric about the line through the axis of rotation and the center of the hole for attaching one end of the spring, only one coordinate of the center of mass need be calculated. If the flat leaf is considered to consist of three component parts--a semicircle, a triangle, and a circular hole--the y component of the center of mass is

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \tag{A-1}$$

and each mass is

$$m_i = \rho h A_i \tag{A-2}$$

ρ being the density of the leaf material, h the thickness, and A is here the area of the leaf parts. The ρh cancel from numerator and denominator. The mass of the shaft and any spacer used will be ignored. Then the y component of the center of mass is

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} \tag{A-3}$$

$$= \frac{\left(\frac{\pi R_1^2}{2}\right)\left(\frac{-4R_1}{3\pi}\right) + (R_1 c)\left(\frac{c}{3}\right) + (-\pi r_3^2)(-d)}{\frac{\pi R_1^2}{2} + R_1 c + (-\pi r_3^2)}$$

$$\bar{y} = \frac{-\frac{2}{3}(0.199)^3 + \frac{0.199\,(0.082)^2}{3} + \pi(0.022)^2\,(0.121)}{\frac{\pi}{2}(0.199)^2 + (0.199)\,(0.082) - \pi(0.022)^2}$$

or

$$\bar{y} = \left[\frac{-0.00462}{0.077}\right] = -0.0600 \text{ in.} \tag{A-4}$$

This distance is the effective torque lever arm of the leaf.

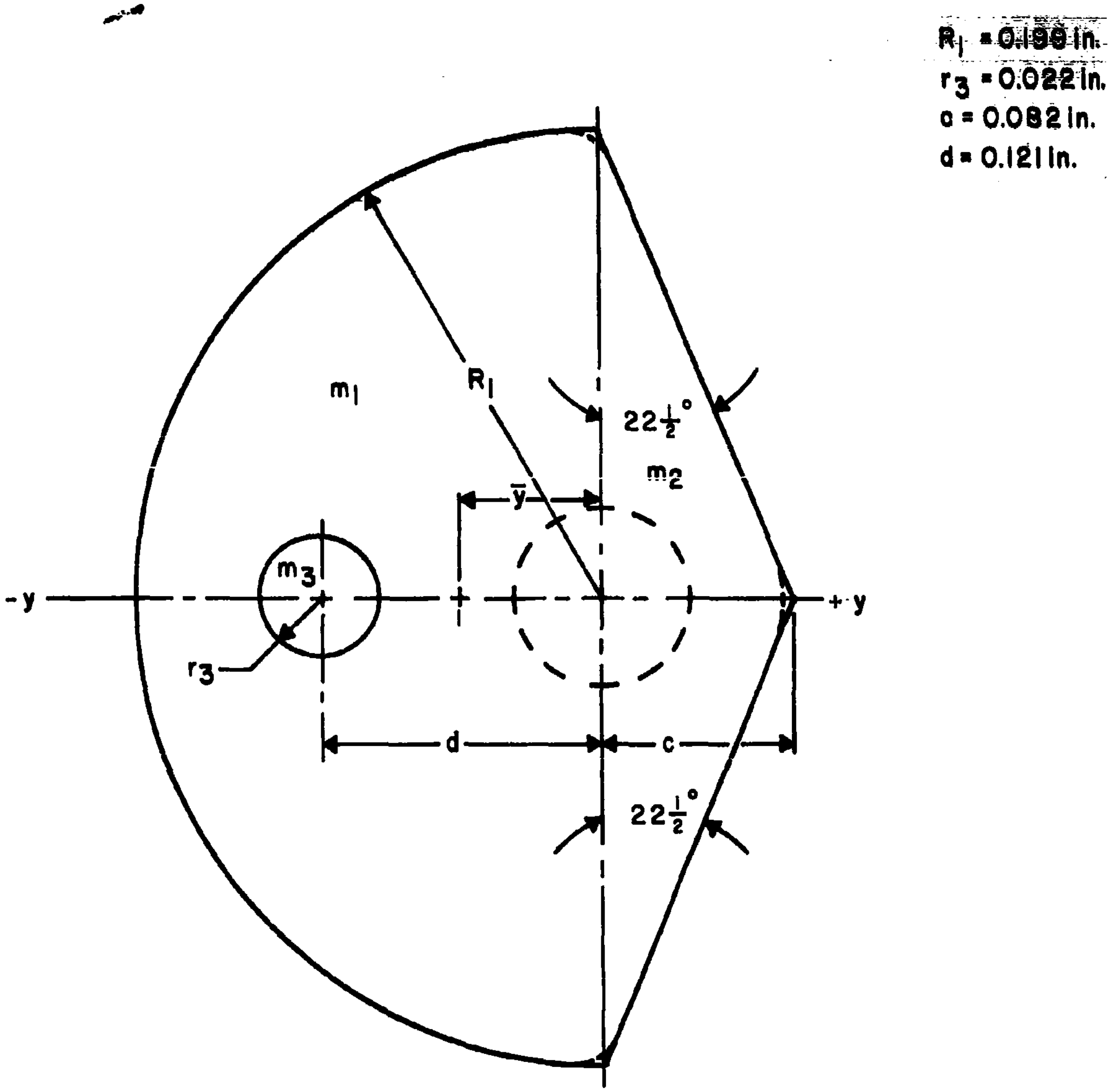

Figure A-1. The center of mass of a leaf.

The moment of inertia is obtained by dividing the leaf surface into five geometric areas, as shown in figure A-2. The moment of inertia of each component part may be expressed in terms of its mass and radius of gyration k

$$I_i = m_i k_i^2 = \rho h A_i k_i^2 \tag{A-5}$$

The moment of inertia about the axis of rotation is then

$$I = I_1 + I_2 + I_3 + I_4 + I_5 \tag{A-6}$$

$$I = \rho h \Big[\left(\frac{\pi R_1^2}{2}\right) \left(\frac{R_1^2}{2}\right) + \left(\frac{\pi R_2^2}{8}\right)\left(\frac{R_2^2}{2}\right) + (-\pi r_3^2)\left(\frac{r_3^2}{2} + d^2\right)$$

$$+ 2\left(\frac{ab}{2}\right) \left(\frac{a^2}{18} + \frac{b^2}{18} + \frac{4a^2}{9} + \frac{b^2}{9}\right)\Big] \tag{A-7}$$

where the radii of gyration are found in handbooks. The parallel-axes theorem was employed to obtain the polar moments of inertia of circular area 3 and triangular areas 4 and 5 about the leaf axis of rotation.

When numerical values are substituted, the radius of gyration squared of the leaf is found to be $k^2 = I/\rho hA$

$$k^2 = \frac{\rho h}{\rho h(0.077)} \Big[\frac{\pi}{4} (0.199)^4 + \frac{\pi}{16} (0.076)^4 - \pi(0.022)^2 \left(\frac{0.022^2}{2} + 0.121^2\right)$$

$$+ (0.076)(0.184)\left(\frac{0.076^2}{2} + \frac{0.184^2}{6}\right)\Big]$$

$$k^2 = \frac{1}{0.077} [0.001232 + 0.000\ 006 - 0.000\ 023 + 0.000\ 119]$$

or

$$k^2 = 0.0173 \text{ in.}^2 \tag{A-8}$$

The values of the leaf constants calculated in this Appendix are not to be considered related to the values employed in the main body of the report, but only serve as illustrative examples.

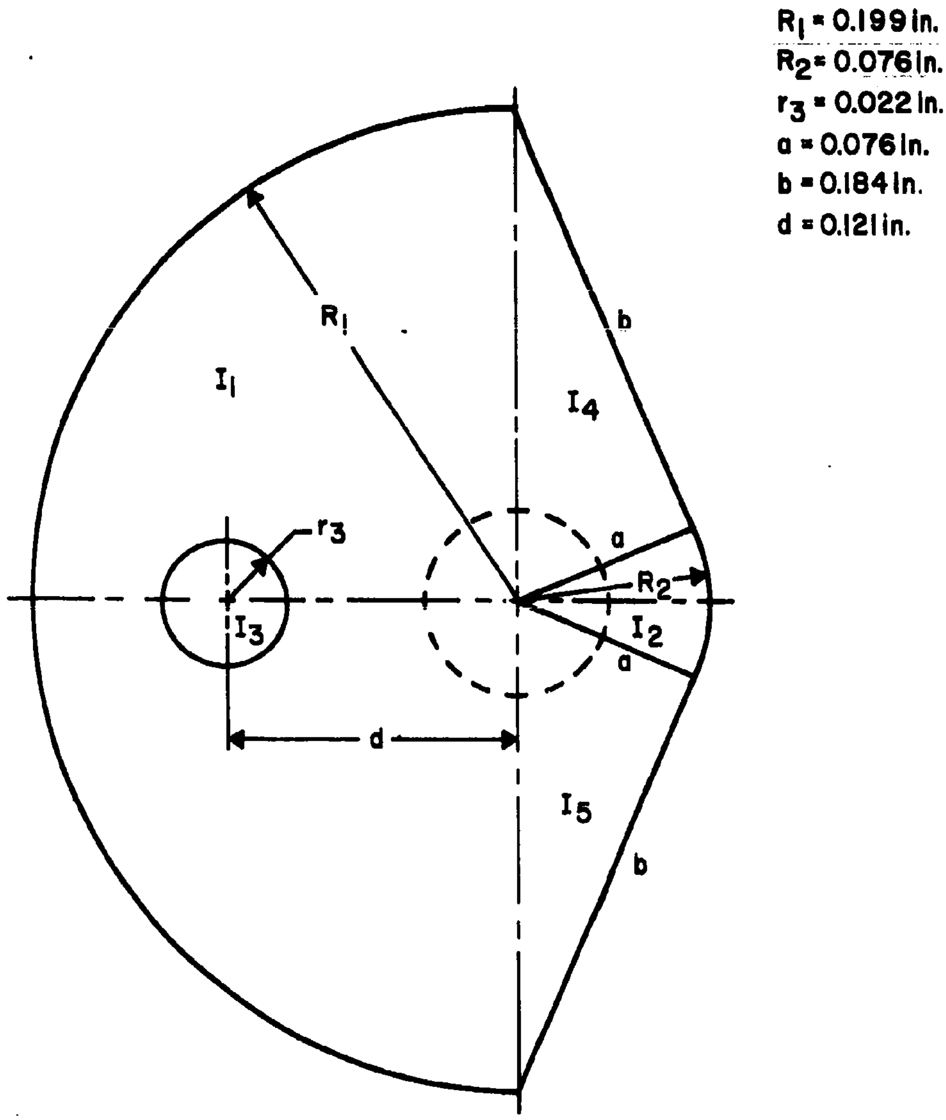

Figure A-2. Moment of inertia of leaf.

APPENDIX B

RESPONSE TO INCREASING AND DECREASING LINEAR ACCELERATIONS

The response of the leaf-spring system to both increasing and decreasing linear accelerations will now be derived. Consider a class of linear accelerations starting at zero time and continuing until time t is equal to $\frac{\psi}{\beta}$, where $0 < \psi \leq \pi$. If all linear functions pass through the point $(\frac{\psi}{2\beta}, \bar{A})$, the velocity change, or area, for all accelerations will be the same, as seen in figure B-1. The equation of a straight line passing through a point $(\frac{\psi}{2\beta}, \bar{A})$ is

$$A(t) = \bar{A} + b(t - \frac{\psi}{2\beta});\ 0 \leq t \leq \frac{\psi}{\beta} \qquad \text{(B-1)}$$

where the constant slope b may be positive, zero, or negative; and $\bar{A}$ is the average acceleration.

If this acceleration function is substituted in the equation of motion (14) for the leaf with zero friction, the equation becomes

$$\frac{d^2\theta}{dt^2} + \beta^2\theta = h(t)\frac{myg}{I}[(\bar{A} - N_o) + b(t - \frac{\psi}{2\beta})] \qquad \text{(B-2)}$$

where β is the natural frequency of oscillation. The initial conditions are:
at $t = 0$, $\theta = 0$ and $\frac{d\theta}{dt} = 0$. To simplify the algebra, let

$$P = \frac{mygb}{I};\quad Q = \frac{myg(\bar{A} - N_o)}{I} \qquad \text{(B-3)}$$

Then (B-2) reduces to

$$\frac{d^2\theta}{dt^2} + \beta^2\theta = h(t)\ [Q + P(t - \frac{\psi}{2\beta})] \qquad \text{(B-4)}$$

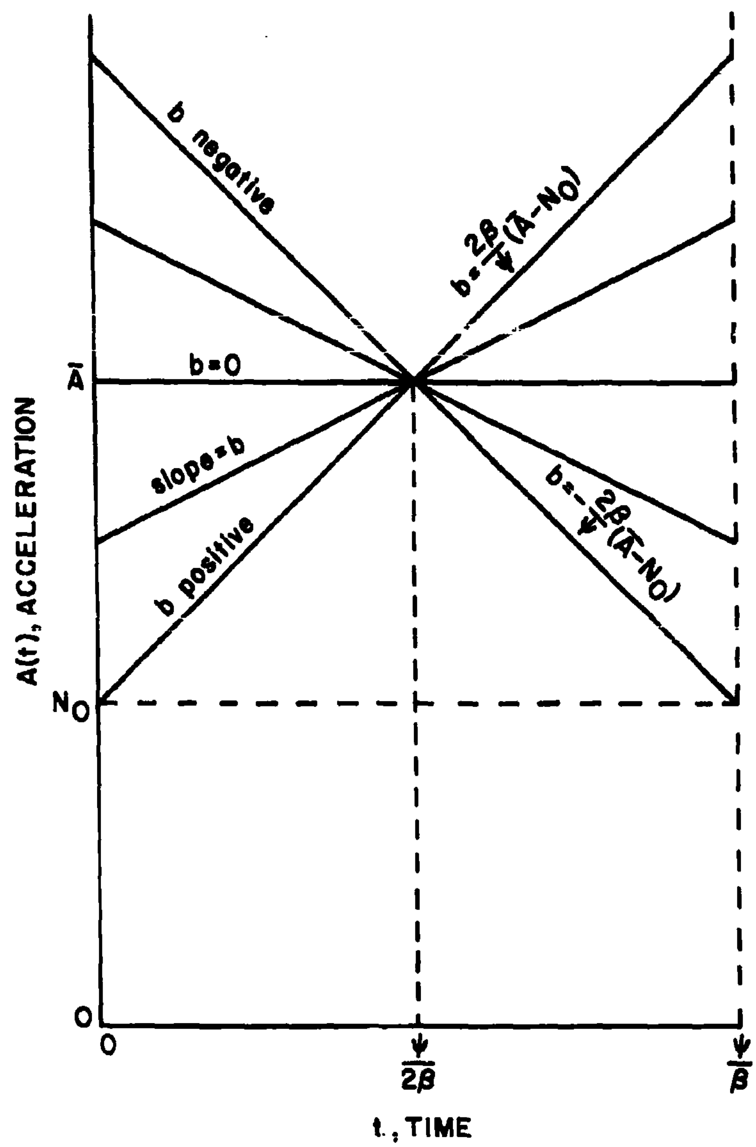

Figure B-1. Linear accelerations of the same velocity change.

The Laplace transform is

$$(s^2 + \beta^2)\,\Theta = \frac{Q - \frac{P\psi}{2\beta}}{s} + \frac{P}{s^2},$$

where Θ is the transform of θ

When this is divided by $(s^2 + \beta^2)$ and the inverse transform is obtained, the solution to (B-4) is

$$\theta = h(t)\left[\frac{(Q - \frac{P\psi}{2\beta})}{\beta^2}(1 - \cos\beta t) + \frac{Pt}{\beta^2} - \frac{P}{\beta^3}\sin\beta t\right] \tag{B-5}$$

After time $\beta t = \psi$, when the velocity change is the same for all values of b, $\theta = \theta_v$

$$\theta_v = \frac{Q - \frac{P\psi}{2\beta}}{\beta^2}(1 - \cos\psi) + \frac{P\psi}{\beta^3} - \frac{P}{\beta^3}\sin\psi$$

$$\theta_v = \frac{Q}{\beta^2}(1 - \cos\psi) + \frac{P}{\beta^3}\left(\frac{\psi}{2} + \frac{\psi}{2}\cos\psi - \sin\psi\right) \tag{B-6}$$

For all values of ψ such that $0 < \psi < \pi$, the coefficient of P is negative. Therefore, the angle of rotation of the leaf will be greater for negative values of P (negative slopes b); the more negative P (and b), the greater the rotation. To limit the term in brackets in (B-2) and (B-4) to positive values only, it is necessary to restrict b, and therefore P, such that

$$-\frac{2\beta}{\psi}(\bar{A} - N_o) \leq b \leq \frac{2\beta}{\psi}(\bar{A} - N_o)$$

This agrees with Hausner's conclusion that the greatest leaf rotation is obtained when most of the acceleration is applied at the start.

Figure B-2 is a plot of (B-5) for the case of $\psi = \pi$ for negative, positive, and zero sloping acceleration functions. For

$b = +\frac{2\beta}{\pi}(\bar{A} - N_o)$, 0, and $-\frac{2\beta}{\pi}(\bar{A} - N_o)$, respectively, (B-5)

becomes

$$\theta = \frac{2Q}{\beta^2}\,\frac{(\beta t - \sin\beta t)}{\pi} \tag{B-7a}$$

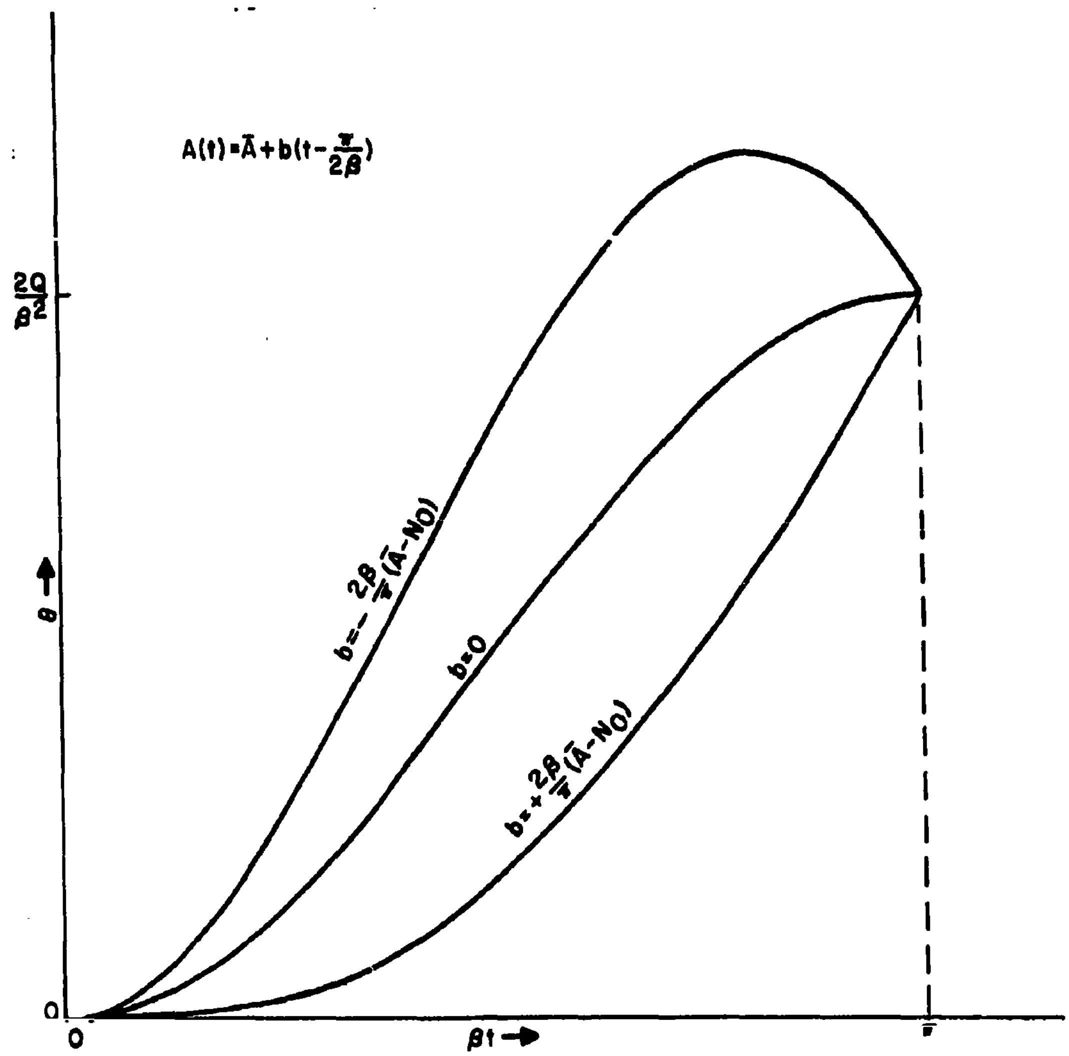

Figure B-2. Response of leaf to linear applied accelerations.

$$\theta = \frac{2Q}{\beta^2} \frac{(1 - \cos \beta t)}{2} \qquad \text{(B-7b)}$$

$$\theta = \frac{2Q}{\beta^2} \left[1 - \cos \beta t - \frac{(\beta t - \sin \beta t)}{\pi} \right] \qquad \text{(B-7c)}$$

It is seen from these curves that when $t = \frac{\pi}{\beta}$, $\theta = \frac{2Q}{\beta^2}$ in all cases. However, for all cases of negative P, θ rises rapidly to $\frac{2Q}{\beta^2}$ at an earlier time, continues to a maximum, and then returns to $\frac{2Q}{\beta^2}$ on the rebound.

APPENDIX C

THE VELOCITY CHANGE TO ARM A LEAF

The velocity change, or time integral of applied acceleration, which a leaf must receive to reach its arming position, will be derived for two simple acceleration functions. The first function considered will be the step function lasting as long as it takes the leaf to arm. Then a rectangular pulse of acceleration will be used, which lasts just long enough to supply sufficient momentum to the leaf by the time the pulse ends to cause it to continue rotating to its arming angle.

Case I

For the first case considered, a constant acceleration A is suddenly applied at zero time to a frictionless leaf-spring system whose opposing spring force is equal to a constant. Thus, in the equation of motion (14), $\lambda = 0$, $t_o = 0$, $C_1 = 1$, $C_2 = 1$, and

A(t) [or more exactly A(t) cos θ] is a constant A.

$$\frac{d^2\theta}{dt^2} = h(t)\,\frac{mgy}{I}\,(A - N_o)\,;\ \theta_o = \dot{\theta}_o = 0 \qquad \text{(C-1)}$$

In all cases, the initial values of the angle of rotation θ and its angular velocity will be taken as zero.

The solution of this equation is obtained by the use of Laplace transforms. The transform of equation (C-1) is, letting $L\,[\theta] = \Theta$

$$s^2\ \Theta = \frac{mgy}{I}\,(A - N_o)\,\frac{1}{s}.$$

When this is divided by s^2, and the inverse transform is taken, the solution becomes

$$\theta = \frac{mgy\ (A - N_o)\ t^2}{2I} \qquad \text{(C-2)}$$

The leaf will rotate to its arming angle θ_r after a time t_r. Therefore

$$t_r = \left(\frac{2I\theta_r}{myg(A - N_o)}\right)^{\frac{1}{2}} \qquad \text{(C-3)}$$

In this first case, it will be noted that θ increases with t^2 and the angular velocity

$$\frac{d\theta}{dt} = \frac{mgy(A - N_o)}{I} t \tag{C-4}$$

increases linearly with time as long as the acceleration continues constant.

The velocity change applied until the leaf rotates to θ_r is then the time integral of the acceleration, or just the product of the constant acceleration and time to arm:

$$V = A\, gt_r = \left(\frac{2gI\theta_r A^2}{my(A-N_o)}\right)^{\frac{1}{2}} \tag{C-5}$$

<u>Case II</u>

The next case considered is the same as the first except that now the acceleration lasts only long enough to give the leaf sufficient momentum to reach θ_r as the angular velocity drops to zero. (If the leaf reached θ_r with an excess of velocity, the acceleration pulse could have been terminated earlier and the leaf would still have armed.) The acceleration is now a rectangular pulse initiated at $t = 0$ and terminated at some $t = t_1$:

$$A(t) \cos\theta = A\, [h(t)-h(t-t_1)] \tag{C-6}$$

The equation of motion (14) is then

$$\frac{d^2\theta}{dt^2} = \frac{mygA}{I} [h(t)-h(t-t_1)] - \frac{mygN_o}{I} \tag{C-7}$$

The transform of this equation is

$$s^2\, \Theta \;=\; \frac{mygA}{Is} (1 - e^{-t_1 s}) - \frac{mygN_o}{Is} .$$

This is divided by s^2 and rearranged to give

$$\Theta \;=\; \frac{myg}{I} \left(\frac{A - N_o}{s^3} - \frac{A\, e^{-t_1 s}}{s^3}\right).$$

The inverse transform is then:

$$\theta = \frac{mgy}{I}\left[\frac{(A - N_O)t^2}{2} - h(t - t_1)\frac{A}{2}(t - t_1)^2\right] \qquad \text{(C-8)}$$

The differentiation of (C-8) results in the following equation for the angular velocity, since the time derivative of the step function is the delta function:

$$\frac{d\theta}{dt} = \frac{myg}{I}\left[(A - N_o)t - h(t - t_1)\,A(t - t_1) - \frac{A}{2}(t - t_1)^2\delta(t-t_1)\right]$$

By definition

$$(t - t_1)\,\delta(t - t_1) = 0 \qquad \text{(C-9)}$$

so that the last term of (C-8) is zero.

$$\frac{d\theta}{dt} = \frac{myg}{I}\left[(A - N_o)t - h(t - t_1)\,A(t - t_1)\right] \qquad \text{(C-10)}$$

It is noted that the angular velocity in this case rises linearly until time t_1 with a slope $(A - N_o)$ and then falls linearly with a slope $(-N_o)$. The angle of rotation θ increases as before with the time squared, but at time t_1 the curve undergoes inflexion, rising less rapidly to a peak.

For the area under the acceleration pulse, or velocity change, to be a minimum, the required boundary conditions are that, at

$t = t_r$, $\frac{d\theta}{dt} = 0$ and $\theta = \theta_r$, where $t_r > t_1$ so that $h(t_r - t_1) = 1$

$$\theta_r = \frac{myg}{I}\left[\frac{(A - N_O)}{2}t_r^2 - \frac{A}{2}(t_r - t_1)^2\right]$$

$$0 = \frac{myg}{I}\left[(A - N_O)t_r - A(t_r - t_1)\right]$$

The second equation can be solved for t_r:

$$t_r = \frac{A}{N_o}\,t_1, \qquad \text{(C-11)}$$

and the result substituted in the first.

$$\theta_r = \frac{myg}{2I} \frac{A(A - N_o)}{N_o} t_1^2 \tag{C-12}$$

The length of time which the acceleration pulse must endure for the leaf to rotate an angle θ_r is therefore

$$t_1 = \left(\frac{2I\theta_r N_o}{myg\ A(A - N_o)}\right)^{\frac{1}{2}} \tag{C-13}$$

while the time it takes the leaf to arm is

$$t_r = \left(\frac{2I\theta_r A}{myg\ N_o(A-N_o)}\right)^{\frac{1}{2}} \tag{C-14}$$

The velocity change which the leaf receives is then

$$V = A\ gt_1 = \left(\frac{2Ig\theta_r A\ N_o}{my\ (A-N_o)}\right)^{\frac{1}{2}} \tag{C-15}$$

The above results were first obtained by Hausner, and are here included in abbreviated form so that they can be discussed in the context in which they were obtained in the main body of this report and compared with the following new derivations.

Case III

The two above cases provide useful results, but for a better approximation it is desirable to determine what the effect is of replacing the constant opposing spring force by a more realistic linear spring force. This requires retaining the term $\lambda\theta/I$ in the equation of motion, (14), so that the opposing-spring force increases as the leaf rotates. Therefore, the first two cases are now re-derived with a linear spring force. For the case of a step function acceleration applied with a linear spring the equation of motion (14) is

$$\frac{d^2\theta}{dt^2} + \frac{\lambda\theta}{I} = h(t)\ \frac{myg}{I}\ (A - N_o) \tag{C-16}$$

The Laplace transform is

$$s^2\ \Theta + \frac{\lambda}{I}\ \Theta = \frac{myg(A - N_o)}{Is}$$

If this equation is divided by $s^2 + \frac{\lambda}{I}$, and the inverse transform is taken, the solution for (C-16) is found to be

$$\theta = \frac{myg(A - N_o)}{\lambda} \left(1 - \cos \sqrt{\frac{\lambda}{I}}\; t\right) \qquad \text{(C-17)}$$

It is noted that θ, instead of increasing indefinitely with the time-squared as in the case of a constant spring force, now increases as a one-minus-the-cosine (or sine-squared) function of time reaching a maximum angle of rotation

$$\theta_m = \frac{2myg(A - N_o)}{\lambda} \qquad \text{(C-18)}$$

in a time

$$t_m = \sqrt{\frac{I}{\lambda}}\; \pi \qquad \text{(C-19)}$$

Thus the arming time can be no greater than (C-19) and is in general equal to

$$t_r = \sqrt{\frac{I}{\lambda}} \cos^{-1}\left[1 - \frac{\lambda\theta_r}{myg(A - N_o)}\right] \qquad \text{(C-20)}$$

The angular velocity is a sinusoidal function of time.

$$\frac{d\theta}{dt} = \frac{myg(A - N_o)}{\sqrt{\lambda I}} \sin \sqrt{\frac{\lambda}{I}}\; t \qquad \text{(C-21)}$$

The velocity change received until the leaf arms is now

$$V = Agt_r = Ag \sqrt{\frac{I}{\lambda}} \cos^{-1}\left[1 - \frac{\lambda\theta_r}{myg(A - N_o)}\right] \qquad \text{(C-22)}$$

Case IV

Now consider the case of the rectangular pulse acceleration modified by a linear spring opposing force. The equation of motion is (C-7) with the addition of the term $\lambda\theta/I$.

$$\frac{d^2\Theta}{dt^2} + \frac{\lambda\Theta}{I} = \frac{mygA}{I}\left[h(t) - h(t-t_1)\right] - \frac{mygNo}{I} \qquad \text{(C-23)}$$

The transform of this equation is

$$s^2\ \Theta + \frac{\lambda\Theta}{I} = \frac{mygA}{Is}\ (1 - e^{-t_1 s}) - \frac{mygN_o}{Is}$$

When this is divided by $s^2 + \frac{\lambda}{I}$, and the inverse transform obtained, the solution of (C-23) is found to be

$$\Theta = \frac{myg}{\lambda}\left[(A - N_o)(1 - \cos\sqrt{\frac{\lambda}{I}}\,t) - A\left\{1 - \cos\sqrt{\frac{\lambda}{I}}\,(t - t_1)\right\}h(t-t_1)\right] \qquad \text{(C-24)}$$

The angular velocity is obtained by differentiating, recognizing that

$$\left[1 - \cos\sqrt{\frac{\lambda}{I}}\,(t - t_1)\right]\delta(t - t_1) = 0.$$

$$\frac{d\Theta}{dt} = \frac{myg}{\sqrt{\lambda I}}\left[(A - N_o)\sin\sqrt{\frac{\lambda}{I}}\ t - A\left\{\sin\sqrt{\frac{\lambda}{I}}\,(t - t_1)\right\}h(t - t_1)\right] \qquad \text{(C-25)}$$

The conditions for minimum velocity change are, as in Case II, that, at $t = t_r$, $\Theta = \Theta_r$ and $\frac{d\Theta}{dt} = 0$, where $t_r > t_1$.

$$\Theta_r = \frac{myg}{\lambda}\left[(A - N_o)(1 - \cos\sqrt{\frac{\lambda}{I}}\,t_r) - A\left\{1 - \cos\sqrt{\frac{\lambda}{I}}\,(t_r - t_1)\right\}\right]$$

$$0 = \frac{myg}{\sqrt{\lambda I}}\left[A - N_o)\sin\sqrt{\frac{\lambda}{I}}\ t_r - A\sin\sqrt{\frac{\lambda}{I}}\,(t_r - t_1)\right]$$

The second equation can be solved for $\sqrt{\frac{\lambda}{I}}\ t_r$ in terms of $\sqrt{\frac{\lambda}{I}}\ t_1$.

$$\tan\sqrt{\frac{\lambda}{I}}\ t_r = \frac{\sin\sqrt{\frac{\lambda}{I}}\,t_1}{\cos\sqrt{\frac{\lambda}{I}}\,t_1 - \frac{(A - N_o)}{A}} \qquad \text{(C-26)}$$

When this result is substituted in the first equation, a solution can be obtained for t_1. However, in order to simplify the algebra, it is useful to first rearrange the equation for Θ_r as

follows:

$$\frac{\lambda\theta_r}{mygA} + \frac{N_o}{A} = \cos\sqrt{\frac{\lambda}{I}}\,(t_r - t_1) - \frac{(A - N_o)}{A}\cos\sqrt{\frac{\lambda}{I}}\,t_r$$

$$= \left[\cos\sqrt{\frac{\lambda}{I}}\,t_1 - \frac{(A - N_o)}{A}\right]\cos\sqrt{\frac{\lambda}{I}}\,t_r + \sin\sqrt{\frac{\lambda}{I}}\,t_1\,\sin\sqrt{\frac{\lambda}{I}}\,t_r$$

or

$$\frac{mygN_o + \lambda\theta_r}{mygA} = \left[\cos\sqrt{\frac{\lambda}{I}}\,t_1 - \frac{(A - N_o)}{A}\right]\cos\sqrt{\frac{\lambda}{I}}\,t_r\left[1 + \frac{\sin\sqrt{\frac{\lambda}{I}}\,t_1}{\cos\sqrt{\frac{\lambda}{I}}\,t_1 - \frac{(A-N_o)}{A}}\tan\sqrt{\frac{\lambda}{I}}\,t_r\right]$$

If (C-26) is now substituted, and the trigonometric identity

$$1 + \tan^2\sqrt{\frac{\lambda}{I}}\,t_r = \frac{1}{\cos^2\sqrt{\frac{\lambda}{I}}\,t_r}$$

employed, the equation reduces to

$$\frac{mygN_o + \lambda\theta_r}{mygA} = \frac{\cos\sqrt{\frac{\lambda}{I}}\,t_1 - \frac{(A - N_o)}{A}}{\cos\sqrt{\frac{\lambda}{I}}\,t_r}$$

$$= \left(\sin^2\sqrt{\frac{\lambda}{I}}\,t_1 + \left[\cos\sqrt{\frac{\lambda}{I}}\,t_1 - \frac{(A - N_o)}{A}\right]^2\right)^{\frac{1}{2}}$$

$$= \left(1 + \frac{(A - N_o)^2}{A^2} - \frac{2(A - N_o)}{A}\cos\sqrt{\frac{\lambda}{I}}\,t_1\right)^{\frac{1}{2}}$$

A solution for $\cos\sqrt{\frac{\lambda}{I}}\,t_1$ can be obtained by squaring both sides of this equation.

$$\cos \sqrt{\frac{\lambda}{I}}\, t_1 = \frac{1 + \left(\frac{A - N_o}{A}\right)^2 - \left(\frac{mygN_o + \lambda\theta_r}{mygA}\right)^2}{2\,\frac{A - N_o}{A}} \qquad \text{(C-27)}$$

The velocity change which the leaf receives is now

$$V = Agt_1 = Ag\sqrt{\frac{I}{\lambda}} \cos^{-1} \frac{1 + \left(\frac{A - N_o}{A}\right)^2 - \left(\frac{mygN_o + \lambda\theta_r}{mygA}\right)^2}{2\left(\frac{A - N_o}{A}\right)} \qquad \text{(C-28)}$$

<u>Case V</u>

The last case to be considered will be that of an applied delta function of infinite amplitude lasting an infinitely short time with a minimum velocity change V. The opposing spring force will be considered linear. The acceleration is

$$gA(t) = V\delta(t) \qquad \text{(C-29}$$

and the equation of motion (14) becomes

$$\frac{d^2\theta}{dt^2} + \frac{\lambda\theta}{I} = \frac{my}{I} V\delta(t) - \frac{mygN_o}{I} \qquad \text{(C-30)}$$

The transform of this equation is

$$s^2\,\Theta + \frac{\lambda}{I}\,\Theta = \frac{myV}{I} - \frac{mygN_o}{Is}$$

When this is divided by $s^2 + \frac{\lambda}{I}$, and the inverse transform taken, the solution of (C-30) is found to be

$$\theta = \frac{myV}{\sqrt{\lambda I}} \sin\sqrt{\frac{\lambda}{I}}\, t - \frac{mygN_o}{\lambda}\left(1 - \cos\sqrt{\frac{\lambda}{I}}\, t\right)$$

or

$$\theta = \frac{mygN_o}{\lambda}\left[\left(\frac{\lambda V^2}{IN_o^2 g^2} + 1\right)^{\frac{1}{2}} \sin\left(\sqrt{\frac{\lambda}{I}}\, t + \phi\right) - 1\right] \;;\; \phi = \tan^{-1} \frac{gN_o}{V}\sqrt{\frac{I}{\lambda}} \qquad \text{(C-31)}$$

The time required for θ to reach its maximum angle of rotation

$$\theta_m = \frac{mygN_o}{\lambda}\left[\left(\frac{\lambda V^2}{Ig^2 N_o^2} + 1\right)^{\frac{1}{2}} - 1\right] \qquad \text{(C-32)}$$

is seen to be

$$t_{min} = \sqrt{\frac{I}{\lambda}}\left(\frac{\pi}{2} - \tan^{-1} \frac{N_o g}{V} \sqrt{\frac{I}{\lambda}}\right)$$

or, since $\frac{\pi}{2} - \tan^{-1} x = \tan^{-1} \left(\frac{1}{x}\right)$ (C-33)

$$t_{min} = \sqrt{\frac{I}{\lambda}} \tan^{-1}\left(\frac{V}{gN_o} \sqrt{\frac{\lambda}{I}}\right)$$

If (C-32) is solved for the velocity change V in terms of the angle of rotation $\theta_m = \theta_r$, it is found that

$$V = gN_o \sqrt{\frac{I}{\lambda}}\left(\left[1 + \frac{\lambda\theta_r}{mygN_o}\right]^2 - 1\right)^{\frac{1}{2}}$$

or

$$V = \left(\frac{I\theta_r N_o g}{my}\left[2 + \frac{\lambda\theta_r}{mygN_o}\right]\right)^{\frac{1}{2}} \qquad \text{(C-34)}$$

The angular velocity of rotation obtained by differentiating (C-31) is

$$\frac{d\theta}{dt} = \frac{mygN_o}{\sqrt{\lambda I}}\left(1 + \frac{\lambda V^2}{IN_o^2 g^2}\right)^{\frac{1}{2}} \cos\left(\sqrt{\frac{\lambda}{I}}\, t + \phi\right) \qquad \text{(C-35)}$$

or

$$\frac{d\theta}{dt} = \frac{my}{I}\left(V^2 + \frac{I}{\lambda} N_o^2 g^2\right)^{\frac{1}{2}} \cos\left(\sqrt{\frac{\lambda}{I}}\, t + \phi\right)$$

With a step jump in the initial velocity from zero to

$$\left.\frac{d\theta}{dt}\right|_{t=0} = \frac{myV}{I} \qquad \text{(C-36)}$$

APPENDIX D

LEAF ARMING TIME AS A FUNCTION OF ITS RELEASE TIME

Consider the differential equation of motion of the form

$$\frac{d^2\theta}{dt^2} + \beta^2 \theta = h(t - t_o) \left[P(e^{-at} - e^{-bt}) - Q\right]; \ \theta_o = \dot{\theta}_o = 0 \qquad \text{(D-1)}$$

where

$$\text{for } t = t_o, \ P(e^{-at_o} - e^{-bt_o}) \geq Q \qquad \text{(D-2)}$$

The Laplace transform of this equation can be obtained as

$$s^2 \Theta + \beta^2 \Theta = Pe^{-at_o} \frac{e^{-st_o}}{s + a} - Pe^{-bt_o} \frac{e^{-st_o}}{s + b} - Q \frac{e^{-st_o}}{s}.$$

When this is divided by $s^2 + \beta^2$, the solution of the differential equation is found to be

$$\theta = h(t - t_o) \left\{ Pe^{-at_o}\left[\frac{1}{a^2 + \beta^2} e^{-a(t - t_o)} + \frac{1}{\beta(a^2 + \beta^2)^{\frac{1}{2}}} \sin(\beta t - \phi_a)\right] \right.$$

$$- Pe^{-bt_o} \left[\frac{1}{b^2 + \beta^2} e^{-b(t - t_o)} + \frac{1}{\beta(b^2 + \beta^2)^{\frac{1}{2}}} \sin(\beta t - \phi_b)\right]$$

$$\left. - \frac{Q}{\beta^2} \left[1 - \cos \beta(t - t_o)\right] \right\}$$

where

$$\tan \phi_a = \frac{\beta}{a}; \ \tan \phi_b = \frac{\beta}{b}$$

This is now rearranged by expanding each sin $(\beta t-\phi)$ and factoring out $\frac{P}{\beta^2}$.

$$\theta = h(t-t_o)\frac{P}{\beta^2}\Bigg\{e^{-at_o}\left[\frac{\beta^2}{a^2+\beta^2}e^{-a(t-t_o)}+\frac{a\beta}{a^2+\beta^2}\sin\beta(t-t_o)-\frac{\beta^2}{a^2+\beta^2}\cos\beta(t-t_o)\right]$$

$$-e^{-bt_o}\left[\frac{\beta^2}{b^2+\beta^2}e^{-b(t-t_o)}+\frac{b\beta}{b^2+\beta^2}\sin\beta(t-t_o)-\frac{\beta^2}{b^2+\beta^2}\cos\beta(t-t_o)\right]$$

$$-\frac{Q}{P}\left[1-\cos\beta(t-t_o)\right]\Bigg\} \tag{D-3}$$

After a time $t = t_r$, θ becomes equal to the angle of release for the leaf, or $\theta = \theta_r$

$$\theta_r = \frac{P}{\beta^2}\Bigg\{e^{-at_o}\left[\frac{\beta^2}{a^2+\beta^2}e^{-a(t_r-t_o)}+\frac{a\beta}{a^2+\beta^2}\sin\beta(t_r-t_o)-\frac{\beta^2}{a^2+\beta^2}\cos\beta(t_r-t_o)\right]$$

$$-e^{-bt_o}\left[\frac{\beta^2}{b^2+\beta^2}e^{-b(t_r-t_o)}+\frac{b\beta}{b^2+\beta^2}\sin\beta(t_r-t_o)-\frac{\beta^2}{b^2+\beta^2}\cos\beta(t_r-t_o)\right]$$

$$-\frac{Q}{P}\left[1-\cos\beta(t_r-t_o)\right]\Bigg\} \tag{D-4}$$

The length of time it takes the leaf to arm is $t_r - t_o$, which will be called T.

$$\theta_r = \frac{P}{\beta^2}\Bigg\{e^{-at_o}\left[\frac{\beta^2}{a^2+\beta^2}e^{-aT}+\frac{a\beta}{a^2+\beta^2}\sin\beta T-\frac{\beta^2}{a^2+\beta^2}\cos\beta T\right]$$

$$-e^{-bt_o}\left[\frac{\beta^2}{b^2+\beta^2}e^{-bT}+\frac{b\beta}{b^2+\beta^2}\sin\beta T-\frac{\beta^2}{b^2+\beta^2}\cos\beta T\right]$$

$$-\frac{Q}{P}\left[1-\cos\beta T\right]\Bigg\} \tag{D-5}$$

Equation (D-5) is seen to present the arming time T as an implicit function of the release time t_o, all other symbols representing constants. The derivative of an implicit function.

$$\theta_r = f(T, t_o) \tag{D-6}$$

is given by

$$\frac{dT}{dt_o} = -\frac{\frac{\partial f}{\partial t_o}}{\frac{\partial f}{\partial T}} \tag{D-7}$$

for the values of the variables t_o, T such that

$$\frac{\partial f}{\partial T} \neq 0 \tag{D-8}$$

where $\frac{\partial f}{\partial t_o}$ and $\frac{\partial f}{\partial T}$ are partial derivatives of (D-5)

$$\frac{\partial f}{\partial t_o} = -ae^{-at_o}\left[\frac{\beta^2}{a^2+\beta^2}\, e^{-aT} + \frac{a\beta}{a^2+\beta^2}\sin \beta T - \frac{\beta^2}{a^2+\beta^2}\cos \beta T\right]$$

$$+be^{-bt_o}\left[\frac{\beta^2}{b^2+\beta^2}\, e^{-bT} + \frac{b\beta}{b^2+\beta^2}\sin \beta T - \frac{\beta^2}{b^2+\beta^2}\cos \beta T\right]$$

$$\frac{\partial f}{\partial T} = e^{-at_o}\left[-\frac{a\beta^2}{a^2+\beta^2}\, e^{-aT} + \frac{a\beta^2}{a^2+\beta^2}\cos \beta T + \frac{\beta^3}{a^2+\beta^2}\sin \beta T\right]$$

$$-e^{-bt_o}\left[\frac{-b\beta^2}{b^2+\beta^2}\, e^{-bT} + \frac{b\beta^2}{b^2+\beta^2}\cos \beta T + \frac{\beta^3}{b^2+\beta^2}\sin \beta T\right]$$

$$-\frac{\beta Q}{P}\sin \beta T$$

When these expressions for $\frac{\partial f}{\partial t_o}$ and $\frac{\partial f}{\partial T}$ are substituted in (D-7), the total derivative of T with respect to t_o is found to be, after cancelling β,

$$\frac{dT}{dt_o} = \frac{e^{-at_o}\left[q_2 e^{-aT} + q_1 \sin \beta T - q_2 \cos \beta T\right] - e^{-bt_o}\left[q_5 e^{-bT} + q_4 \sin \beta T - q_5 \cos \beta T\right]}{e^{-at_o}\left[-q_2 e^{-aT} + q_3 \sin \beta T + q_2 \cos \beta T\right] - e^{-bt_o}\left[-q_5 e^{-bT} + q_6 \sin \beta T + q_5 \cos \beta T\right] - \frac{Q}{P}\sin \beta T} \tag{D-9}$$

where

$$q_1 = \frac{a^2}{a^2+\beta^2}; \qquad q_2 = \frac{a\beta}{a^2+\beta^2}; \qquad q_3 = \frac{\beta^2}{a^2+\beta^2}$$

$$q_4 = \frac{b^2}{b^2+\beta^2}; \qquad q_5 = \frac{b\beta}{b^2+\beta^2}; \qquad q_6 = \frac{\beta^2}{b^2+\beta^2}$$

APPENDIX E

ANALYTICAL EXPRESSIONS FOR GUN ACCELERATION-TIME CURVES

The determination of an analytical expression for gun accelerations as a function of time is a special type of curve fitting problem. The usual curve-fitting task is related to finding an equation that gives the best fit to a set of points, or perhaps a specific curve. The conatants of the equation are calculated but once, sometimes to considerable precision. However, when, instead of a single curve, a wide variety of curves of varying shapes must be expressed in the same analytic form, the method of solution is not so straightforward. One technique is to search for functions whose general shapes are that of the given curves; and then to obtain as much information as possible about these functions to aid in finding methods of determining the parameters for each curve. Another technique is to plot the given curves on various kinds of logarithmic paper to determine if any relationships can be determined between the variables and their logarithms. For example, a straight-line segment on semilogarithmic paper indicates an exponential relationship. Hausner used this technique to obtain a sum of three exponentials for a close fit to the acceleration-time curve of the 81 mm, T28E6 mortar at one increment.

A representative set of gun acceleration curves is shown in figure (E-1). The curves are difficult for engineers to obtain accurately, and no simple theoretical derivations have been made for the accelerations obtained in gun firings. It will be noted that all the curves are of a type that rise sharply to a rounded peak and drop off more slowly in what appears to be an exponential manner. In addition, all curves have an inflection point beyond the peak, and some--particularly those with high increment power--have an inflection point also before the peak. Actually, for this setback-leaf problem, only the upper part of the curves need be fitted to an analytic expression since the leaves do not begin to move until the acceleration exceeds the equivalent spring acceleration N_o, and arming is completed before the curve has dropped below, say, one-third of the peak. Therefore, it will be assumed that any fit of the upper two-thirds of the curves is sufficient.

Among the functions that have the same general shape as the curves of figure (E-1) are the skewed normal distribution.

$$f(t) = A_o t^a \; e^{\frac{-(t - \bar{t})}{2\sigma^2}}$$

where $\bar{t}$ is the mean and σ^2 is the variance, and the gamma distribution

$$f(t) = A_o t^a \; e^{-bt}$$

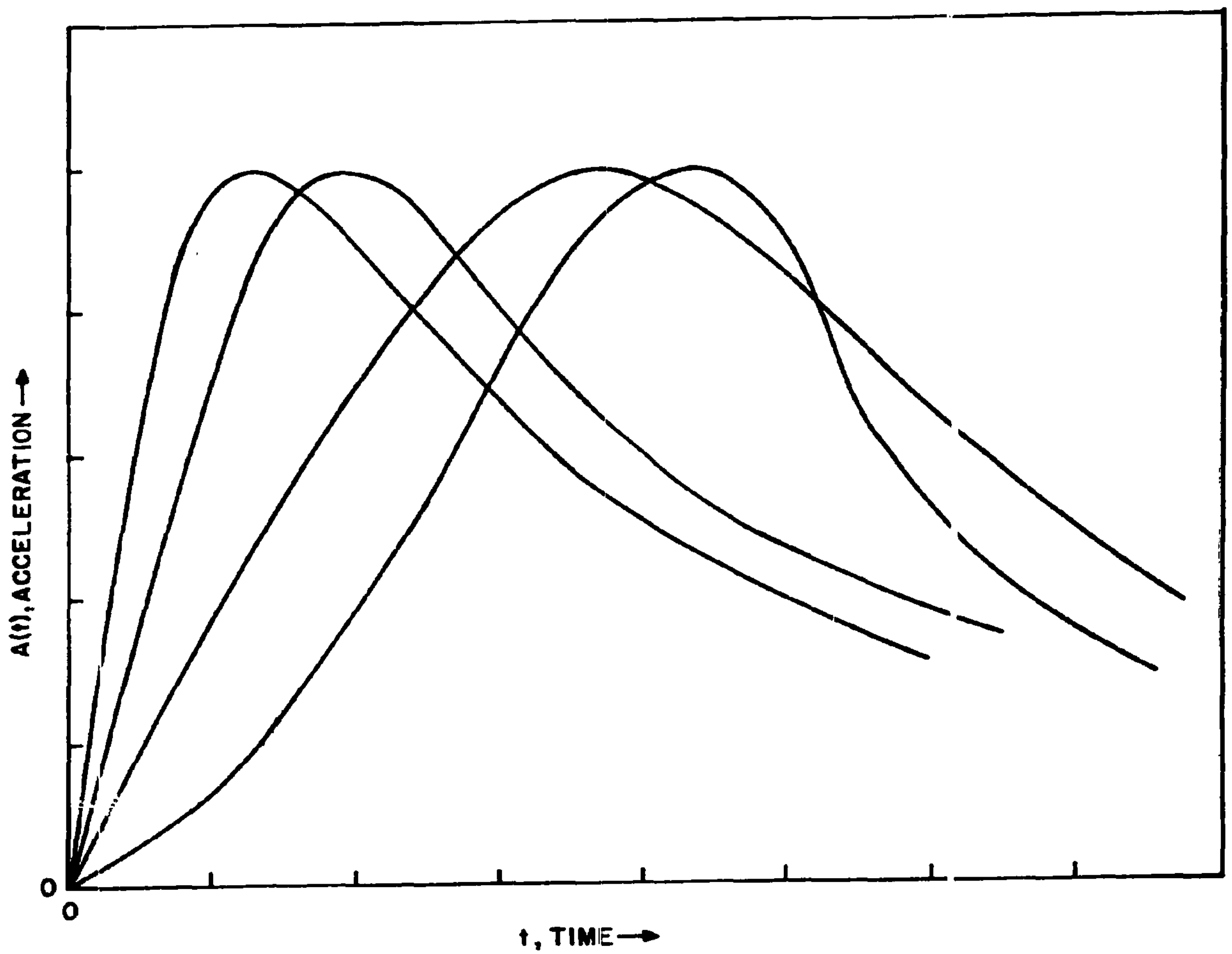

Figure E-1. Typical gun acceleration-time curves.

Although good fits can be obtained for an individual curve with either of these functions, it is a laborious process, and no systematic technique could be devised for determining the parameters for each of the curves. In addition (and this was the most important reason these functions were not used), when the acceleration is expressed in terms of one of these functions, the analytic solutions of equation(14) are complicated and difficult to do much with. Furthermore, they require the use of a large number of analog computer components in obtaining machine solutions.

For these reasons, other functions were sought to represent the family of gun curves. The sum of several exponentials seemed to be the most promising. Before these are discussed, it is helpful to examine the exponential function of time:

$$A = A_o e^{-at} \qquad \text{(E-1)}$$

By taking the natural logarithm of both sides,

$$\ln A = \ln A_o - at$$

(where ln is used to indicate the logarithm to the base e), it is seen that an exponential function will plot as a straight line on semilogarithmic graph paper with A_o the intercept with the log axis and a slope equal to $(-a)$. Another method of determining the value of "a" results from the fact that, when $t = \frac{1}{a}$, $A = \frac{A_o}{e}$. Thus, "a" is equal to the inverse of the time coordinate of the point on the line where A is equal to $\frac{1}{e}$ times the intercept A_o.

Hausner used the following sum of three exponentials to fit the acceleration curve of the 81-mm T28E6 mortar shown in figure E-2:

$$A = 2700\, e^{-172.7t} - 3330\, e^{-451.4t} + 630\, e^{-1084.5t}\ g \qquad \text{(E-2)}$$

where t is in seconds.

This is obtained by plotting the given gun curve on the semi-log paper and fitting a straight line to the approximately straight exponential trailing edge to obtain the constants of the first exponential. Then, the difference between the straight line and the plot of the gun curve is also plotted on the same paper. Another straight line is fitted to the right-hand portion of this difference curve to obtain the constants of the second exponential. The difference is plotted again, and the process continues until the

remaining difference becomes small. However, this is more difficult than it might appear. The first difference curve is usually quite curved; therefore, it may not be possible to find a very long portion to which a straight line can be fitted. This results in the next difference being an odd shaped curve, and the whole process can break down. The resulting sum of exponentials is then not a good representation of most acceleration curves, even though Hausner was able to find a close fit for his particular curve. In addition, the sum of three exponentials to represent a set of acceleration curves meets the same objections as the gamma distribution and the skewed normal distribution in that there appears to be no systematic and simple way of varying the parameters of the expression as the curves vary. Already, there are several parameters in the equation of motion, and an additional four or five greatly complicates any theoretical analysis, since it is desirable to know how results are affected as each parameter is varied.

The sum of three exponentials has five parameters in the form:

$$A(t) = A_1 e^{-at} - (A_1 + A_2)\, e^{-bt} + A_2\, e^{-ct} \qquad \text{(E-3)}$$

Now, it will be shown that this expression can be rearranged to a more usable form

$$A(t) = f_1(t) - f_2(t) = A_1(e^{-at} - e^{-bt}) - A_2(e^{-bt} - e^{-ct})\,; \quad a < b < c, \quad A_1 > A_2 \qquad \text{(E-4)}$$

This is recognized as the difference of two similar functions, identical except for their parametric values. In figure E-2, Hausner's curve is shown split into these two component parts. The interesting feature of these two curves is that they both have the same general shape as the gun curves. This suggests the possibility that one of these curves alone could be used to represent the gun curves; it will be shown that, fortunately, this is true.

It should be mentioned first, however, that if it is necessary to find a close fit to a given gun curve, expressions in the form of (E-4) show promise. This was not pursued further, but it can be seen that the larger curve given by the first part of (E-4) should be selected to peak before the given gun curve, with its amplitude adjusted, so that the gun curve fits beneath with the trailing edges coincident. To prevent the difference curve from going negative at the start, it is necessary that the initial slope of the larger curve be greater than the smaller. From the differentiation of (E-4), it is seen that the slopes of the two curves are:

$$\frac{df_1}{dt} = A_1(be^{-bt} - ae^{-at})$$

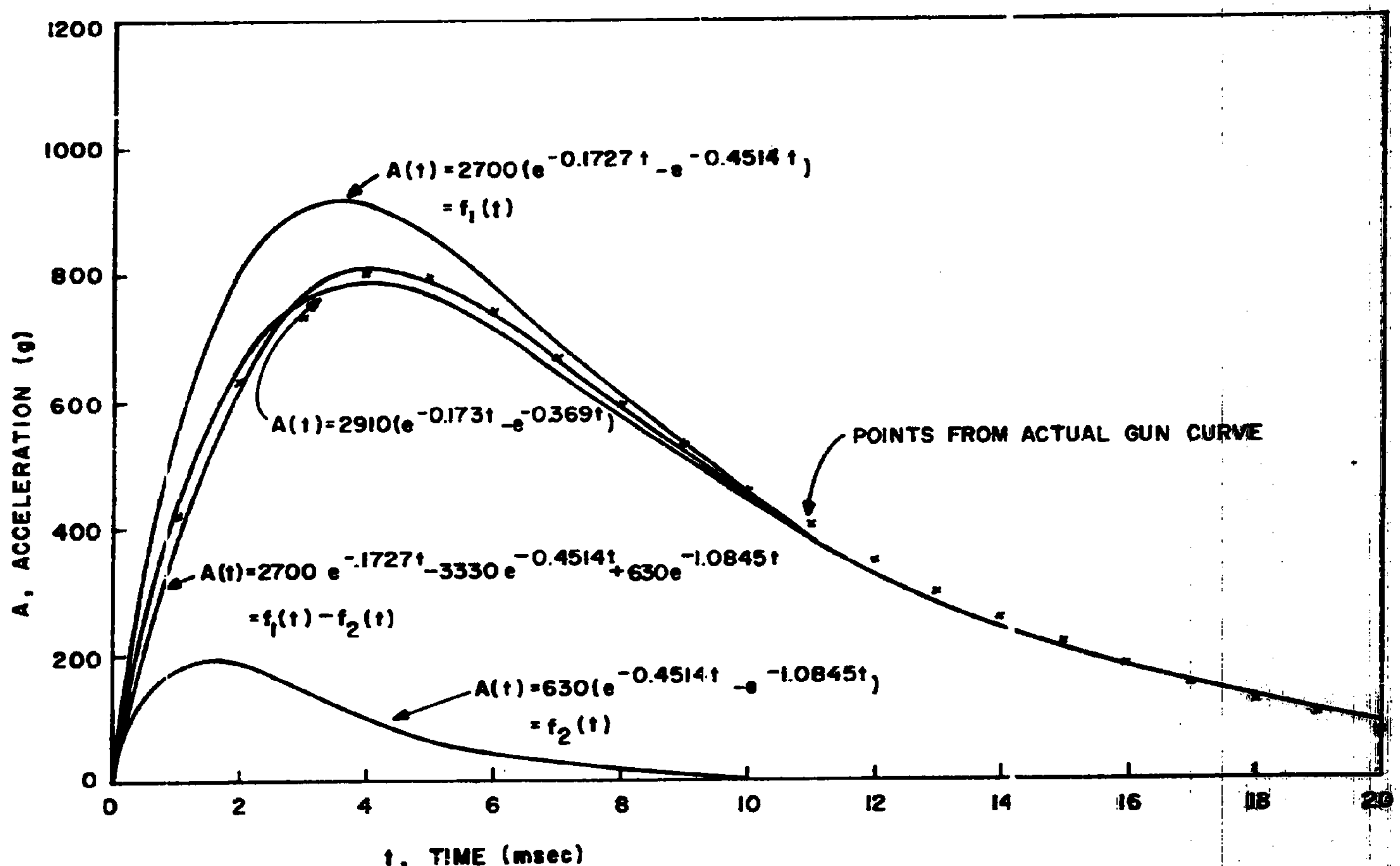

Figure E-2. Analytical functions for matching gun accelerations.

and

$$\frac{df_2}{dt} = A_2 \ (ce^{-ct} - be^{-bt}).$$

Therefore, it is required that at t = 0,

$$A_1 \ (b-a) > A_2 \ (c-b)$$

for the slope of the larger curve to be the greater. Also, there would be more flexibility if all four of the exponential coefficients were different; i.e., if the first exponential coefficient of f_2 is not b, but some other constant.

Now, an examination will be made of the two-exponent exponential curve that is part of equation (E-4)

$$f = A_o \ (e^{-at} - e^{-bt}); \ b > a \qquad \text{(E-5)}$$

A_o is only an amplitude or scale factor that can be set equal to unity. For b greater than a, this function is positive for all values of t greater than zero. The curve always has a single peak that is followed by an inflection point. The time at which these points occur can be obtained from the first and second derivatives of (E-5)

$$\frac{df}{dt} = -ae^{-at} + be^{-bt} \qquad \text{(E-6)}$$

$$\frac{d^2f}{dt^2} = a^2e^{-at} - b^2e^{-bt} \qquad \text{(E-7)}$$

Setting each derivative equal to zero and solving for "at", it is found that the times at which the maximum and inflection points occur, respectively, are

$$at_m = \frac{1}{(b/a)-1} \ln \frac{b}{a} = \ln \left(\frac{b}{a}\right)^{\frac{1}{(b/a)-1}} \qquad \text{(E-8)}$$

$$at_{in} = \frac{2}{(b/a)-1} \ln \frac{b}{a} = 2 \ at_m \qquad \text{(E-9)}$$

(It should be noted that such solutions cannot be obtained analytically for the functions discussed earlier in this appendix.) The time at which the curve undergoes inflection is seen to be double the peak time, so that the drop from the peak always takes longer than the rise.

When (E-8) and (E-9) are substituted into (E-5), it is found that the values of the function at the peak and at the inflection point are, respectively:

$$f_m = e^{-at_m} - e^{-\frac{b}{a}(at_m)} = \left(\frac{b}{a}\right)^{\frac{-1}{\frac{b}{a}-1}} - \left(\frac{b}{a}\right)^{\frac{-b/a}{\frac{b}{a}-1}}$$

$$= \left(\frac{b}{a}\right)^{\frac{-b/a}{b/a-1}} \left(\frac{b}{a} - 1\right) \qquad \text{(E-10)}$$

and

$$f_{in} = e^{-at_{in}} - e^{-\frac{b}{a}(at_{in})} = \left(\frac{b}{a}\right)^{\frac{-2}{\frac{b}{a}-1}} - \left(\frac{b}{a}\right)^{\frac{-2\frac{b}{a}}{\frac{b}{a}-1}}$$

$$= \left(\frac{b}{a}\right)^{\frac{-2\frac{b}{a}}{\frac{b}{a}-1}} \left[\left(\frac{b}{a}\right)^2 - 1\right] \qquad \text{(E-11)}$$

From (E-8) - (E-11), it is seen that at_m, at_{in}, f_m, and f_{in} are all functions of b/a alone. Let η equal the ratio b/a, which is always larger than unity, and the equations then become:

$$at_m = \frac{1}{\eta-1} \ln\eta \qquad \text{(E-12)}$$

$$at_{in} = \frac{2}{\eta-1} \ln\eta \qquad \text{(E-13)}$$

$$f_m = \frac{\eta-1}{\eta^{\left(\frac{\eta}{\eta-1}\right)}} \qquad \text{(E-14)}$$

$$f_{in} = \frac{\eta^2 - 1}{\eta^{\left(\frac{2\eta}{\eta-1}\right)}} \qquad \text{(E-15)}$$

The equations (E-12),(E-14) and (E-15) are plotted in figure E-3. It will be noted that the maximum possible value of at_m is unity as η approaches unity. Therefore, t_m must be less than 1/a. If it should be necessary that t_m be greater than 1/a, the time origin of the curve can be shifted.

The function (E-5) can be expressed in normalized form if it is divided by (E-14) and "at" is replaced by the dimensionless time τ.

$$\frac{f}{f_m} = \frac{e^{-at} - e^{-\frac{b}{a}(at)}}{\dfrac{\eta - 1}{\eta^{\left(\frac{\eta}{\eta - 1}\right)}}} = \frac{\eta^{\left(\frac{\eta}{\eta-1}\right)}}{\eta-1}\left(e^{-\tau} - e^{-\eta\tau}\right) \qquad \text{(E-16)}$$

This normalized function is shown plotted in figure E-4 for values of η equal to 1.1, 1.5, 2, 3, 4, 6, 8, 10, and 20. These curves show how the shape of the function varies as η changes, and aid in the selection of a suitable value of η to fit the gun curve. As an additional aid, these curves are replotted in figure E-5 with their time axes shifted to make the peaks coincident.

To fit a given gun curve to this function, it is necessary to first determine "a" from a plot on semilogarithmic graph paper as described earlier, and then to scale the amplitude of the gun curve to have the same peak as the curves of figures E-4 and E-5 with a new normalized time scale "at." It may be necessary to make adjustments in the value of "a" to get the best fit, and to interpolate between the curves of figures E-4 and E-5 to get the best value of η. These parameters and the amplitude factor A_o should be selected, so that the resulting function gives the best fit to the upper half of the given gun curve. Values of η equal to 2.13 and 10, with "a" equal to 173 per sec, provide two representative curves that are used in the analysis. The choice of η equal to 2.13 results in a curve that gives a good fit to the T28E6 as shown in figure E-2. These two acceleration functions are shown in figure E-6.

In summary, the two-exponent exponential function (E-5) is a useful representation of acceleration curves, since a whole family of gun curves can be approximated by varying a single parameter η, as shown by figure E-4. This function's usefulness is increased, in that it is one of the most tractable mathematical functions to employ in the equations of motion in dynamics.

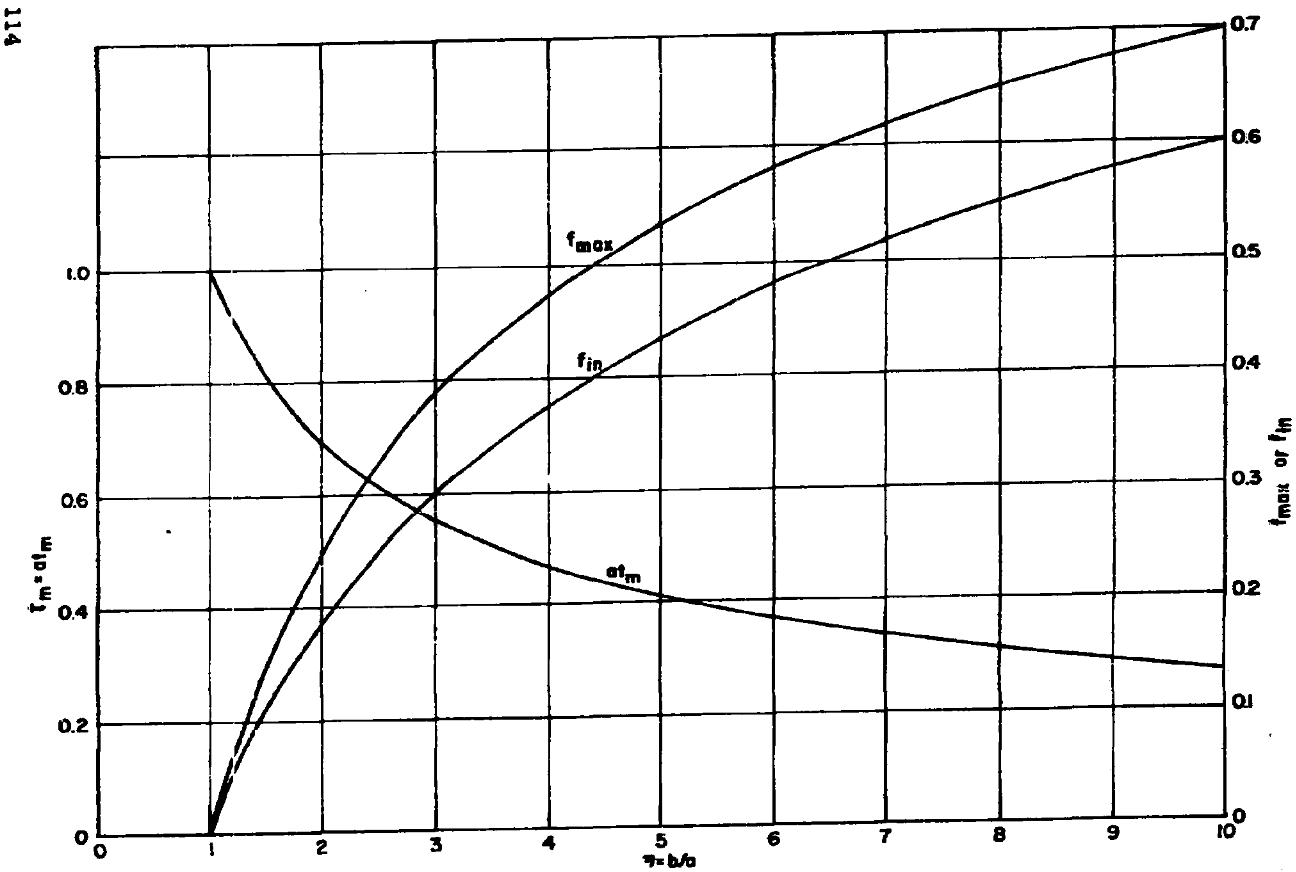

Figure E-3. Plots of at_m, f_{max} and f_{in} versus $\eta = \frac{b}{a}$.

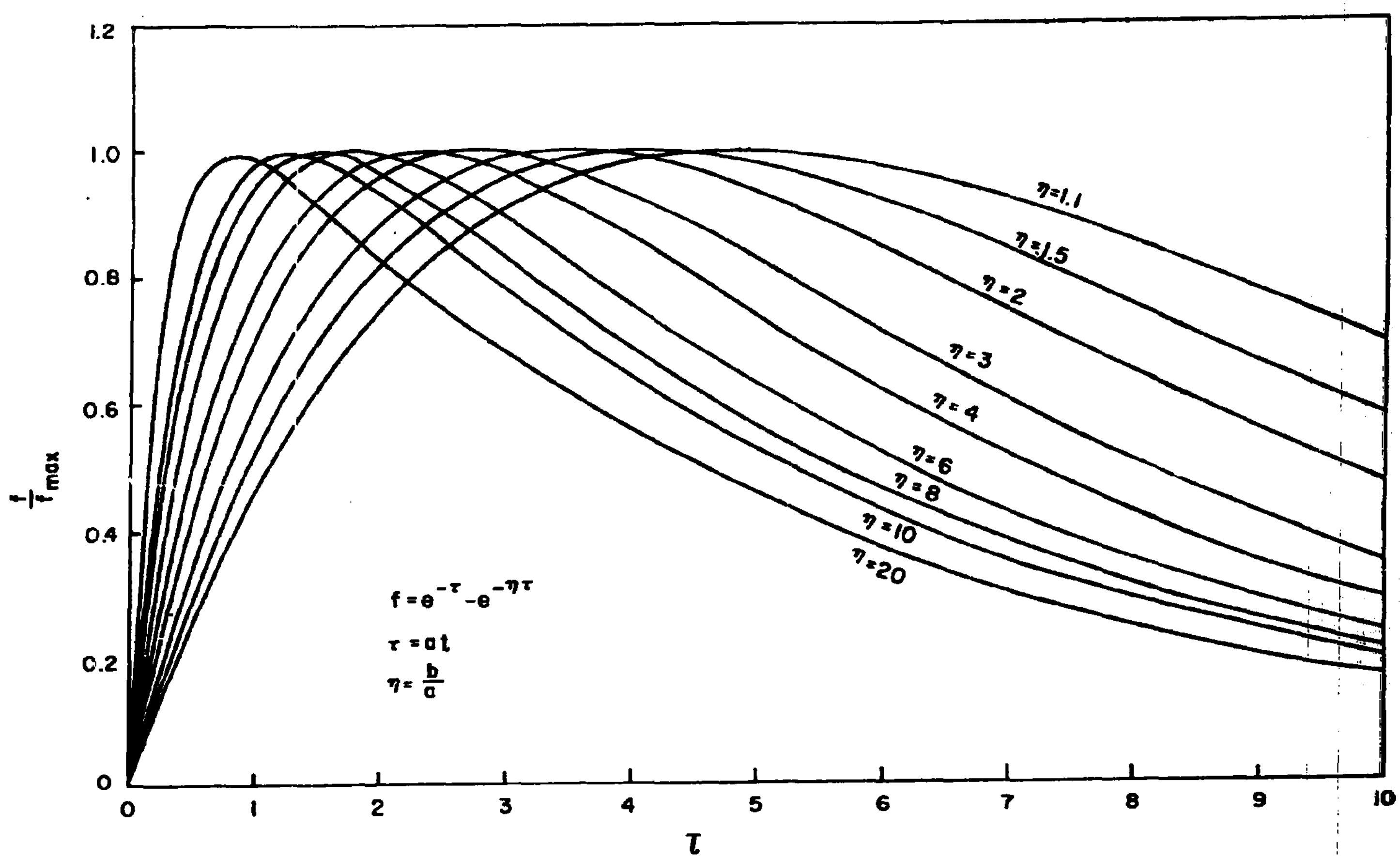

Figure E-4. Dependence of shape acceleration function on η.

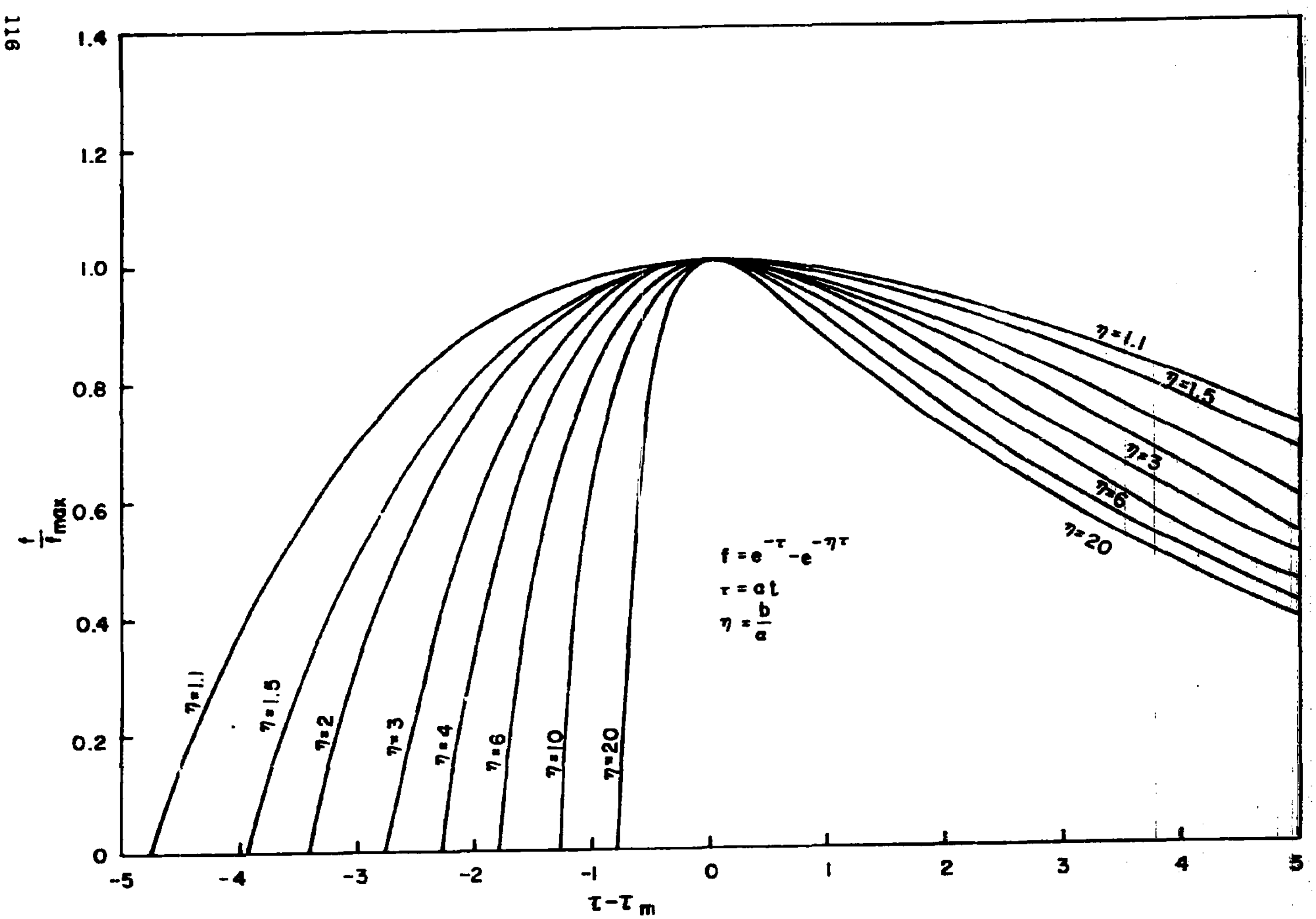

Figure E-5. Curves of previous figure shifted to peak at same time.

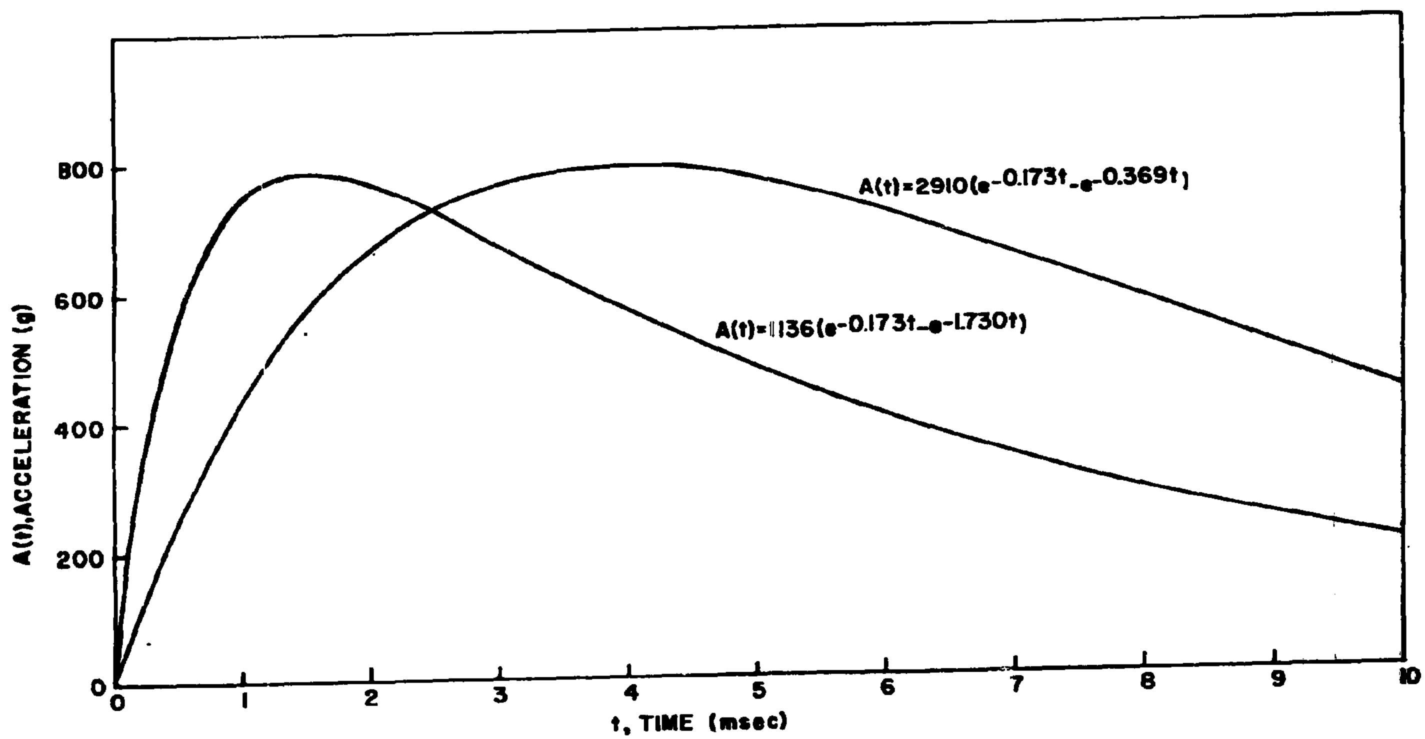

Figure E-6. Acceleration functions used in analysis.

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Harry Diamond Laboratories, Washington, D. C.

ANALYSIS AND DESIGN—ROTARY-TYPE SETBACK LEAF S&A MECHANISMS
William E. Ryan

TR-1190, 11 February 1964, 90 pp text, 32 illus, DA-1P523801A300, AMCMS Code 5522.11.62400, HDL Proj. 46300, UNCLASSIFIED Report

A design procedure is described for obtaining maximum safety and reliability in S&A setback mechanisms of the rotary leaf and spring type. The procedure is based on a mathematical analysis of the operation of setback-mechanisms. The main assumption of the analysis is that friction has an average effective value.

An analytic study is made of the effect of varying the mass of the leaves upon an accidental-drop safety of the mechanism. The drop safety, defined as the absolute minimum velocity change that can arm the device, is derived and evaluated for each set of leaves.

Also included are graphical curves that show the arming time of each leaf of different weight as a function of the time that elapses from moment of firing to their release. The combinations of weights that will arm are readily obtained by adding the arming times together.

The results of this study indicate that leaves of equal weight are almost as safe and reliable as combinations of leaves of varying weights.

1. Setback-leaf systems—Design analysis
2. Safety devices—Mathematical analysis
3. Ordnance projectiles—Safety
4. S&A setback mechanisms, rotary leaf and spring type

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